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Multiple Attribute Group Decision Making with Linguistic Information Based on Linguistic Prioritized Operators

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Author's contribution

The sole author designed, analyzed and interpreted and prepared the manuscript.

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ABSTRACT

In this paper, we propose some linguistic prioritized aggregation operators and uncertain linguistic prioritized aggregation operators for handling the multiple attribute group decision making problems in which the attributes and decision makers are in different priority level. First, we extend the prioritized aggregation (PA) operators [R. R. Yager, Prioritized aggregation operators, International Journal of Approximate Reasoning 48 (2008) 263-274] to linguistic environment and present two linguistic prioritized aggregation operators called the linguistic prioritized weighted average (LPWA) operator and the linguistic prioritized weighted geometric (LPWG) operator. These proposed operators can capture the prioritization phenomenon among the aggregated arguments. Then, some desirable properties of these aggregation operators are examined in detail. We next utilize the proposed operators to develop an approach to multiple attribute group decision making under linguistic environment in which the attributes and decision makers are in different priority level, and a practical example is given to illustrate the practicality and effectiveness of the developed approach.

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1. INTRODUCTION

Multi attribute group decision making (MAGDM) problems are to find the most desirable alternative(s) from a set of feasible alternatives according to the preferences provided by a group of experts [1]. The estimation of the attribute values plays an important role in a MAGDM problem. In the real world, due to the increasing complexity of the socio-economic environment and the vagueness of inherent subjective nature of human thinking, the information about attribute values is usually uncertain or fuzzy [2-5], which leads to the fact that numerical values are inadequate or insufficient to model these information. As a result, it is more suitable to provide the experts' preferences by means of linguistic variables rather than numerical ones. For example, when evaluating the ‘‘comfort’’ or ‘‘design’’ of a car, linguistic terms such as ‘‘good’’, ‘‘medium’’, and ‘‘bad’’ [6] are frequently used, and when evaluating a car's speed, linguistic terms like ‘‘very fast’’, ‘‘fast’’, and ‘‘slow’’ can be used [7]. A linguistic approach is an efficient tool to deal with such situations.

Multi attribute linguistic group decision making consists of choosing the best alternatives according to the linguistic performance values provided and involves the following two phases [8,9]: (1) Aggregation phase of linguistic information: It consists of obtaining a collective linguistic performance value for the alternatives by aggregating the linguistic performance values provided according to all the attributes using the chosen aggregation of linguistic information. (2) Exploitation phase: It consists of establishing a rank ordering among the alternatives according to the collective linguistic performance value for choosing the best alternatives. In the first step, in order to aggregate the individual linguistic preference information into the overall linguistic preference information, linguistic aggregation operators are most widely used. In the past few decades, many scholars have developed a variety of linguistic aggregation operators, which can be classified into the following categories [10]: (1) The linguistic aggregation operators, which are based on linear ordering [11-16]; (2) The linguistic aggregation operators, which are based on the extension principle [17-20] and make computations on the fuzzy numbers that support the semantics of the linguistic labels [21-25]; (3) The linguistic aggregation operators, which are based on symbols [8,26,27], and make computations on the indexes of the linguistic labels; (4) The linguistic aggregation operators, which are based on a 2-tuple fuzzy linguistic representation model [28-36]. This model represents the linguistic assessment information by means of a pair of values called 2-tuples, composed by a linguistic term and a number; and (5) The linguistic aggregation operators, which compute directly with words [10,37-50]. The operators in (1)-(3) develop approximation processes to express the results in the initial expression domain, that produce a consequent loss of information and hence a lack of precision. However those in (4)-(5) allow a continuous representation of the linguistic information on their domains, and therefore can represent any counting of information obtained in an aggregation process without loss of information [10,28,51].

The aforementioned linguistic aggregation operators are usually used to deal with MAGDMs where the attribute and the decision makers are at the same priority level respectively. In these MAGDMs, we have the ability to trade off between attributes. For instance, if C_i and C_j are two attributes with the weights w_i and w_j respectively, then we can compensate for a decrease of q in satisfaction to attribute C_i by gain $\frac{w_j}{w_i}q$ in satisfaction to attribute C_j. However, in many MAGDM problems, this kind of compensation between attributes is infeasible. For example, consider the situation where we are buying a car based on the safety and cost of cars. We should not allow a benefit with respect to cost to compensate for a loss in safety. In this situation we have a typical kind of prioritization of the attributes, i.e., safety has a higher priority than cost. In addition, decision making in a university, president have a higher priority than vice president. Yager [52] first investigated this kind of problem by introducing the prioritized ‘‘and’’ and ‘‘or’’ operators. Then, Yager [53] and Yager et al. [54] gave deeper insights into on this issue. Motivated by the ideas of Yager [52,53] and Yager et al. [54], up to now, a lot of prioritized aggregation operators have been developed in the literature [55-62]. For example, Wei [60] generalized prioritized aggregation operators to hesitant fuzzy environment and developed some hesitant fuzzy prioritized aggregation operators. Furthermore, they applied these operators to develop some models for hesitant fuzzy multiple attribute decision making problems in which the attributes are in different priority level. Yu et al. [61] proposed some interval-valued intuitionistic fuzzy prioritized aggregation operators and investigated the application of these operators in the group decision making under interval-valued intuitionistic fuzzy environment in which the attributes and experts are in different priority level. Yu and Xu [62] investigated the prioritization relationship of attributes in multi-attribute decision making with intuitionistic fuzzy information and developed some prioritized intuitionistic fuzzy aggregation operators by extending the prioritized aggregation operators. However, we are aware that the existing linguistic aggregation operators are difficult to deal with the MAGDM in which the attributes and the decision makers are in different priority level, and there has been rather little work completed for using prioritized aggregation operators to solve the MAGDM with linguistic preference information. To overcome this drawback, in this paper, we develop some linguistic prioritized aggregation operators and uncertain linguistic prioritized aggregation operators. Then, based on these operators, we present some approaches to the MAGDM where the attribute values are linguistic terms, and the attributes and the decision makers are in different priority level. Finally, some practical examples are provided to show the applications of the proposed approaches.

The remainder of this paper is organized as follows. In Section 2, we briefly review the linguistic approach and the prioritized average operator. Section 3 proposes the linguistic prioritized weighted average (LPWA) operator and the linguistic prioritized weighted geometric (LPWG) operator to aggregate the linguistic information. Furthermore, we develop a method for MAGDM based on the proposed operators under linguistic environment. Meanwhile, an example concerning talent introduction is given to demonstrate its practicality and effectiveness of the developed approach. Section 4 gives some concluding remarks.

2. PRELIMINARIES

In this section, we briefly recall the linguistic approach and the prioritized average operator.

2.1 The Linguistic Approach

The linguistic approach is an approximate technique, which represents qualitative aspects as linguistic values by means of linguistic variables. Let $S=\left\{s_i \middle| i=0,1,\cdots,g\right\}$ be a finite and completely ordered discrete linguistic term set with odd cardinality, where s_i represents a possible value for a linguistic variable and $g+1$ is the number of granularity in the linguistic term set. As an illustration, a set of seven terms S could be given as follows [40,45,48,49,63,64,65]:

$$S=\left\{\begin{array}{l} s_0=nothing,\ s_1=very\,low,\ s_2=low,\ s_3=medium, \\ s_4=high,\ s_5=very\,high,\ s_6=perfect \end{array}\right\}$$

Usually, it is required that linguistic term set S should satisfy the following characteristics:

(1) The set is ordered: $s_i \ge s_j$ if $i \ge j$;
(2) There is the negation operator: $neg\left(s_i\right)=s_j$ such that $j=g-i$;
(3) Max operator: $\max\left(s_i,s_j\right)=s_i$ if $s_i \ge s_j$;
(4) Min operator: $\min\left(s_i,s_j\right)=s_i$ if $s_i \le s_j$.

To preserve all the given information, Xu [41,66] extended the discrete linguistic term set S to a continuous linguistic term set $\bar{S}=\left\{s_\alpha \middle| s_0 \le s_\alpha \le s_g, \alpha \in [0,g]\right\}$. If $s_\alpha \in S$, then s_α is called an original linguistic term; otherwise, s_α is called a virtual linguistic term. In general, the decision maker uses the original linguistic terms to evaluate alternatives, and the virtual linguistic terms can only appear in operation.

Considering two linguistic terms $s_\alpha, s_\beta \in \bar{S}$, and $\mu,\mu_1,\mu_2 \in [0,1]$, Xu [41,66] defined some operational laws as follows:

(1) $s_\alpha \oplus s_\beta = s_\beta \oplus s_\alpha = s_{\alpha+\beta}$;
(2) $\mu s_\alpha = s_{\mu\alpha}$;
(3) $\left(\mu_1+\mu_2\right)s_\alpha = \mu_1 s_\alpha \oplus \mu_2 s_\alpha$;
(4) $s_\alpha \otimes s_\beta = s_\beta \otimes s_\alpha = s_{\alpha\beta}$;
(5) $\left(s_\alpha\right)^\mu = s_{\alpha^\mu}$;
(6) $\left(s_\alpha\right)^{\mu_1} \otimes \left(s_\alpha\right)^{\mu_2} = \left(s_\alpha\right)^{\mu_1+\mu_2}$;
(7) $\left(s_\alpha \otimes s_\beta\right)^\mu = \left(s_\alpha\right)^\mu \otimes \left(s_\beta\right)^\mu$.

2.2 Prioritized Average (PA) Operators

The prioritized average (PA) operator was originally introduced by Yager [52,67], which was defined as follows:

Definition 2.1 [52]. Let $C=\{C_1,C_2,\cdots,C_n\}$ be a collection of criteria and that there is a prioritization between the criteria expressed by the linear ordering $C_1 \succ C_2 \succ C_3 \cdots \succ C_n$, indicate criteria C_j has a higher priority than C_k if $j<k$. The value $C_j(x)$ is the performance of any alternative x under criteria C_j, and satisfies $C_j(x)\in[0,1]$. If

$$PA(C_i(x))=\sum_{j=1}^{n} w_j C_j(x), \quad (1)$$

where $w_j=\frac{T_j}{\sum_{j=1}^{n}T_j}$, $T_j=\prod_{k=1}^{j-1}C_k(x)\ (j=2,\cdots,n)$, $T_1=1$. Then PA is called the prioritized average (PA) operator.

3. LINGUISTIC PRIORITIZED AGGREGATION OPERATORS

The prioritized average operator [52,67] has only been used in situations in which the input arguments are the exact values. In this section, we shall investigate the PA operator under linguistic environments. We propose two linguistic prioritized aggregation operators, which can accommodate the situations where the input arguments are linguistic information.

3.1 Linguistic Prioritized Weighted Average (LPWA) Operators

Definition 3.1. Let $\mathrm{LPWA}: \bar{S}^n \to \bar{S}$, if

$$\mathrm{LPWA}(s_{\alpha_1},s_{\alpha_2},\ldots,s_{\alpha_n})=\frac{T_1}{\sum_{j=1}^{n}T_j}s_{\alpha_1}\oplus\frac{T_2}{\sum_{j=1}^{n}T_j}s_{\alpha_2}\oplus\cdots\oplus\frac{T_n}{\sum_{j=1}^{n}T_j}s_{\alpha_n}, \quad (2)$$

where $s_{\alpha_j}\in\bar{S}\ (j=1,2,\ldots,n)$, $T_j=\prod_{k=1}^{j-1}\left(\frac{\alpha_k}{g}\right)\ (j=2,\ldots,n)$, and $T_1=1$, then LPWA is called a linguistic prioritized weighted average (LPWA) operator.

Theorem 3.1. Let $s_{\alpha_j}\in\bar{S}\ (j=1,2,\ldots,n)$, then their aggregated value by using the LPWA operator is also a linguistic term $s_\alpha\in\bar{S}$, and $\alpha=\frac{T_1}{\sum_{j=1}^{n}T_j}\alpha_1+\frac{T_2}{\sum_{j=1}^{n}T_j}\alpha_2+\cdots+\frac{T_n}{\sum_{j=1}^{n}T_j}\alpha_n$.

Proof. According to Definition 3.1, we have

$$\begin{aligned}\mathrm{LPWA}(s_{\alpha_1},s_{\alpha_2},\ldots,s_{\alpha_n})&=\frac{T_1}{\sum_{j=1}^{n}T_j}s_{\alpha_1}\oplus\frac{T_2}{\sum_{j=1}^{n}T_j}s_{\alpha_2}\oplus\cdots\oplus\frac{T_n}{\sum_{j=1}^{n}T_j}s_{\alpha_n}\\&=s_{\frac{T_1}{\sum_{j=1}^{n}T_j}\alpha_1}\oplus s_{\frac{T_2}{\sum_{j=1}^{n}T_j}\alpha_2}\oplus\cdots\oplus s_{\frac{T_n}{\sum_{j=1}^{n}T_j}\alpha_n}\\&=s_{\frac{T_1}{\sum_{j=1}^{n}T_j}\alpha_1+\frac{T_2}{\sum_{j=1}^{n}T_j}\alpha_2+\cdots+\frac{T_n}{\sum_{j=1}^{n}T_j}\alpha_n}\\&=s_\alpha.\end{aligned}$$

Therefore, $\alpha=\frac{T_1}{\sum_{j=1}^{n}T_j}\alpha_1+\frac{T_2}{\sum_{j=1}^{n}T_j}\alpha_2+\cdots+\frac{T_n}{\sum_{j=1}^{n}T_j}\alpha_n$.

Furthermore, we can conclude that

$$\begin{aligned}\min_{1\le i\le n}\{\alpha_i\}&=\frac{T_1}{\sum_{j=1}^{n}T_j}\min_{1\le i\le n}\{\alpha_i\}+\frac{T_2}{\sum_{j=1}^{n}T_j}\min_{1\le i\le n}\{\alpha_i\}+\cdots+\frac{T_n}{\sum_{j=1}^{n}T_j}\min_{1\le i\le n}\{\alpha_i\}\\&\le\alpha=\frac{T_1}{\sum_{j=1}^{n}T_j}\alpha_1+\frac{T_2}{\sum_{j=1}^{n}T_j}\alpha_2+\cdots+\frac{T_n}{\sum_{j=1}^{n}T_j}\alpha_n\\&\le\frac{T_1}{\sum_{j=1}^{n}T_j}\max_{1\le i\le n}\{\alpha_i\}+\frac{T_2}{\sum_{j=1}^{n}T_j}\max_{1\le i\le n}\{\alpha_i\}+\cdots+\frac{T_n}{\sum_{j=1}^{n}T_j}\max_{1\le i\le n}\{\alpha_i\}\\&=\max_{1\le i\le n}\{\alpha_i\}.\end{aligned}$$

Thus, we have $s_{\min_{1\le i\le n}\{\alpha_i\}}\le s_\alpha\le s_{\max_{1\le i\le n}\{\alpha_i\}}$, which implies that $s_\alpha\in\bar{S}$.

By the proof of Theorem 3.1, we can easily obtain the following result:

Theorem 3.2 (Boundedness). Let $s_{\alpha_j}\in\bar{S}\ (j=1,2,\ldots,n)$, then

$$\min_{1\le j\le n}\{s_{\alpha_j}\}\le\mathrm{LPWA}(s_{\alpha_1},s_{\alpha_2},\ldots,s_{\alpha_n})\le\max_{1\le j\le n}\{s_{\alpha_j}\}. \quad (3)$$

Theorem 3.3 (Idempotency). Let $s_{\alpha_j}\in\bar{S}\ (j=1,2,\cdots,n)$. If all $s_{\alpha_j}\ (j=1,2,\cdots,n)$ are equal, i.e., $s_{\alpha_j}=s_\alpha$, for all j, then

$$\mathrm{LPWA}\left(s_{\alpha_1}, s_{\alpha_2}, \ldots, s_{\alpha_n}\right) = s_\alpha . \tag{4}$$

Proof. If $s_{\alpha_j} = s_\alpha$, for all j, then

$$\mathrm{LPWA}\left(s_{\alpha_1}, s_{\alpha_2}, \ldots, s_{\alpha_n}\right) = \mathrm{LPWA}\left(s_\alpha, s_\alpha, \ldots, s_\alpha\right)$$
$$= s_{\frac{T_1}{\sum_{j=1}^n T_j}\alpha + \frac{T_2}{\sum_{j=1}^n T_j}\alpha + \cdots + \frac{T_n}{\sum_{j=1}^n T_j}\alpha} = s_\alpha. \qquad \square$$

Theorem 3.4 (Monotonicity). Let $s_{\alpha_j}, s_{\beta_j} \in \bar{S}$ $(j = 1, 2, \ldots, n)$, if $s_{\alpha_j} \le s_{\beta_j}$, for all j, then

$$\mathrm{LPWA}\left(s_{\alpha_1}, s_{\alpha_2}, \ldots, s_{\alpha_n}\right) \le \mathrm{LPWA}\left(s_{\beta_1}, s_{\beta_2}, \ldots, s_{\beta_n}\right) \tag{5}$$

Proof. This proof for Theorem 3.4 is analogous to the proof for monotonicity of the prioritized average operator in Ref. [52].

3.2 Linguistic Prioritized Weighted Geometric (LPWG) Operators

In the following, we define a linguistic prioritized weighted geometric (LPWG) operator based on the LPWA operator and the geometric mean.

Definition 3.2. Let $\mathrm{LPWG}: \bar{S}^n \to \bar{S}$, if

$$\mathrm{LPWG}\left(s_{\alpha_1}, s_{\alpha_2}, \ldots, s_{\alpha_n}\right) = s_{\alpha_1}^{\frac{T_1}{\sum_{j=1}^n T_j}} \otimes s_{\alpha_2}^{\frac{T_2}{\sum_{j=1}^n T_j}} \otimes \cdots \otimes s_{\alpha_n}^{\frac{T_n}{\sum_{j=1}^n T_j}}, \tag{6}$$

where $s_{\alpha_j} \in \bar{S}$ $(j = 1, 2, \cdots, n)$, $T_j = \prod_{k=1}^{j-1}\left(\frac{\alpha_k}{g}\right)$ $(j = 2, \cdots, n)$, and $T_1 = 1$, then LPWG is called a linguistic prioritized weighted geometric (LPWG) operator.

Theorem 3.5. Let $s_{\alpha_j} \in \bar{S}$ $(j = 1, 2, \ldots, n)$, then their aggregated value by using the LPWG operator is also a linguistic term $s_\alpha \in \bar{S}$, and $\alpha = \alpha_1^{\frac{T_1}{\sum_{j=1}^n T_j}} \alpha_2^{\frac{T_2}{\sum_{j=1}^n T_j}} \cdots \alpha_n^{\frac{T_n}{\sum_{j=1}^n T_j}}$.

Proof. According to Definition 3.2, we have

$$\begin{aligned}\mathrm{LPWG}\left(s_{\alpha_1}, s_{\alpha_2}, \ldots, s_{\alpha_n}\right) &= s_{\alpha_1}^{\frac{T_1}{\sum_{j=1}^n T_j}} \otimes s_{\alpha_2}^{\frac{T_2}{\sum_{j=1}^n T_j}} \otimes \cdots \otimes s_{\alpha_n}^{\frac{T_n}{\sum_{j=1}^n T_j}} \\ &= s_{\alpha_1^{\frac{T_1}{\sum_{j=1}^n T_j}}} \otimes s_{\alpha_2^{\frac{T_2}{\sum_{j=1}^n T_j}}} \otimes \cdots \otimes s_{\alpha_n^{\frac{T_n}{\sum_{j=1}^n T_j}}} \\ &= s_{\alpha_1^{\frac{T_1}{\sum_{j=1}^n T_j}} \alpha_2^{\frac{T_2}{\sum_{j=1}^n T_j}} \cdots \alpha_n^{\frac{T_n}{\sum_{j=1}^n T_j}}} \\ &= s_\alpha.\end{aligned}$$

Therefore, $\alpha = \alpha_1^{\frac{T_1}{\sum_{j=1}^n T_j}} \alpha_2^{\frac{T_2}{\sum_{j=1}^n T_j}} \cdots \alpha_n^{\frac{T_n}{\sum_{j=1}^n T_j}}$. Furthermore, we can conclude that

$$\begin{aligned}\min_{1\le i\le n}\{\alpha_i\} &= \left(\min_{1\le i\le n}\{\alpha_i\}\right)^{\frac{T_1}{\sum_{j=1}^n T_j}} \left(\min_{1\le i\le n}\{\alpha_i\}\right)^{\frac{T_2}{\sum_{j=1}^n T_j}} \cdots \left(\min_{1\le i\le n}\{\alpha_i\}\right)^{\frac{T_n}{\sum_{j=1}^n T_j}} \\ &\le \alpha = \alpha_1^{\frac{T_1}{\sum_{j=1}^n T_j}} \alpha_2^{\frac{T_2}{\sum_{j=1}^n T_j}} \cdots \alpha_n^{\frac{T_n}{\sum_{j=1}^n T_j}} \\ &\le \left(\max_{1\le i\le n}\{\alpha_i\}\right)^{\frac{T_1}{\sum_{j=1}^n T_j}} \left(\max_{1\le i\le n}\{\alpha_i\}\right)^{\frac{T_2}{\sum_{j=1}^n T_j}} \cdots \left(\max_{1\le i\le n}\{\alpha_i\}\right)^{\frac{T_n}{\sum_{j=1}^n T_j}} \\ &= \max_{1\le i\le n}\{\alpha_i\}.\end{aligned}$$

Thus, we have $s_{\min_{1\le i\le n}\{\alpha_i\}} \le s_\alpha \le s_{\max_{1\le i\le n}\{\alpha_i\}}$, which implies that $s_\alpha \in \bar{S}$. $\square$

Similar to Theorems 3.2-3.4, we have the following theorems.

Theorem 3.6 (Boundedness). Let $s_{\alpha_j} \in \bar{S}$ $(j = 1, 2, \ldots, n)$, then

$$\min_{1\le j\le n}\left\{s_{\alpha_j}\right\} \le \mathrm{LPWG}\left(s_{\alpha_1}, s_{\alpha_2}, \ldots, s_{\alpha_n}\right) \le \max_{1\le j\le n}\left\{s_{\alpha_j}\right\}. \tag{7}$$

Theorem 3.7 (Idempotency). Let $s_{\alpha_j} \in \bar{S}$ $(j = 1, 2, \ldots, n)$. If all s_{α_j} $(j = 1, 2, \ldots, n)$ are equal, i.e., $s_{\alpha_j} = s_\alpha$, for all j, then

$$\mathrm{LPWG}\left(s_{\alpha_1}, s_{\alpha_2}, \ldots, s_{\alpha_n}\right) = s_\alpha . \tag{8}$$

Theorem 3.8 (Monotonicity). Let $s_{\alpha_j}, s_{\beta_j} \in \bar{S}$ $(j = 1, 2, \ldots, n)$, if $s_{\alpha_j} \le s_{\beta_j}$, for all j, then

$$\mathrm{LPWG}\left(s_{\alpha_1}, s_{\alpha_2}, \ldots, s_{\alpha_n}\right) \le \mathrm{LPWG}\left(s_{\beta_1}, s_{\beta_2}, \ldots, s_{\beta_n}\right) \tag{9}$$

Lemma 3.1 [68,69]. Let $x_i > 0$, $\lambda_i > 0$, $i = 1, 2, \cdots, n$, and $\sum_{i=1}^n \lambda_i = 1$, then

$$\prod_{i=1}^{n}(x_i)^{\lambda_i} \le \sum_{i=1}^{n}\lambda_i x_i$$

with equality if and only if $x_1 = x_2 = \cdots = x_n$.

Theorem 3.9. Let $s_{\alpha_j} \in \bar{S}$ $(j = 1,2,\ldots,n)$, then we have

$$\mathrm{LPWG}\left(s_{\alpha_1}, s_{\alpha_2}, \ldots, s_{\alpha_n}\right) \le \mathrm{LPWA}\left(s_{\alpha_1}, s_{\alpha_2}, \ldots, s_{\alpha_n}\right).$$

Proof. Let $\mathrm{LPWG}\left(s_{\alpha_1}, s_{\alpha_2}, \ldots, s_{\alpha_n}\right) = s_\alpha$ and $\mathrm{LPWA}\left(s_{\alpha_1}, s_{\alpha_2}, \ldots, s_{\alpha_n}\right) = s_\beta$. By Theorems 3.1 and 3.5, we have $\alpha = \alpha_1^{\frac{T_1}{\sum_{j=1}^n T_j}} \alpha_2^{\frac{T_2}{\sum_{j=1}^n T_j}} \cdots \alpha_n^{\frac{T_n}{\sum_{j=1}^n T_j}}$ and $\beta = \frac{T_1}{\sum_{j=1}^{n} T_j}\alpha_1 + \frac{T_2}{\sum_{j=1}^{n} T_j}\alpha_2 + \cdots + \frac{T_n}{\sum_{j=1}^{n} T_j}\alpha_n$.

Because $\sum_{j=1}^{n}\left(\frac{T_j}{\sum_{j=1}^n T_j}\right) = \frac{\sum_{j=1}^n T_j}{\sum_{j=1}^n T_j} = 1$, by Lemma 3.1, we have

$$\begin{aligned}\alpha &= \alpha_1^{\frac{T_1}{\sum_{j=1}^n T_j}} \alpha_2^{\frac{T_2}{\sum_{j=1}^n T_j}} \cdots \alpha_n^{\frac{T_n}{\sum_{j=1}^n T_j}} \\ &\le \frac{T_1}{\sum_{j=1}^{n} T_j}\alpha_1 + \frac{T_2}{\sum_{j=1}^{n} T_j}\alpha_2 + \cdots + \frac{T_n}{\sum_{j=1}^{n} T_j}\alpha_n \\ &= \beta.\end{aligned}$$

Thus, we can obtain that $s_\alpha \le s_\beta$, i.e., $\mathrm{LPWG}\left(s_{\alpha_1}, s_{\alpha_2}, \ldots, s_{\alpha_n}\right) \le \mathrm{LPWA}\left(s_{\alpha_1}, s_{\alpha_2}, \ldots, s_{\alpha_n}\right)$.

Theorem 3.9 shows that the values obtained by the LPWG operator are not bigger than the ones obtained by the LPWA operator.

3.3 An Approach to Multiple Attribute Group Decision Making with Linguistic Prioritized Aggregation Operators

In this subsection, we utilize the proposed aggregation operators to handle a multiple attribute group decision making with linguistic preference information.

For a group decision making problem with linguistic preference information, let $X = \{x_1, x_2, \cdots, x_m\}$ is the set of alternatives. Let $C = \{c_1, c_2, \ldots, c_n\}$ be a collection of attributes and that there is a prioritization between the attributes expressed by the linear ordering $c_1 \succ c_2 \succ c_3 \succ \cdots \succ c_n$, indicate attribute c_j has a higher priority than c_k if $j < k$. Let $D = \{d_1, d_2, \cdots, d_l\}$ is the set of decision makers and that there is a prioritization between the decision makers expressed by the linear ordering $d_1 \succ d_2 \succ d_3 \succ \cdots \succ d_l$, indicate decision maker d_p has a higher priority than d_q if $p < q$. Suppose that each decision maker provides his own decision matrix $A^{(k)} = \left(s_{\alpha_{ij}^{(k)}}\right)_{m\times n}$ $(k = 1,2,\cdots,l)$, where $s_{\alpha_{ij}^{(k)}} \in \bar{S}$ is a preference value, which takes the form of linguistic variable, given by the decision maker $d_k \in D$, for the alternative $x_i \in X$ with respect to the attribute $c_j \in C$.

In the following, we utilize the LPWA (or LPWG) operator to develop an approach to multi attribute group decision making under a linguistic environment. The main steps can be summarized as follows:

Step 1: Calculate the matrices $T^{(p)} = \left(T_{ij}^{(p)}\right)_{m\times n}$, $(p = 1,2,\cdots,l)$ based on the following equations:

$$T_{ij}^{(p)} = \prod_{k=1}^{p-1}\left(\frac{\alpha_{ij}^{(k)}}{g}\right), \quad p = 2,\cdots,l, \quad i = 1,2,\cdots,m, \quad j = 1,2,\cdots,n, \tag{10}$$

$$T_{ij}^{(1)} = 1, \quad i = 1,2,\cdots,m, \quad j = 1,2,\cdots,n. \tag{11}$$

Step 2: Utilize the LPWA operator:

$$\begin{aligned}s_{\alpha_{ij}} &= \mathrm{LPWA}\left(s_{\alpha_{ij}^{(1)}}, s_{\alpha_{ij}^{(2)}}, \ldots, s_{\alpha_{ij}^{(l)}}\right) \\ &= \frac{T_{ij}^{(1)}}{\sum_{p=1}^{l} T_{ij}^{(p)}} s_{\alpha_{ij}^{(1)}} \oplus \frac{T_{ij}^{(2)}}{\sum_{p=1}^{l} T_{ij}^{(p)}} s_{\alpha_{ij}^{(2)}} \oplus \cdots \oplus \frac{T_{ij}^{(l)}}{\sum_{p=1}^{l} T_{ij}^{(p)}} s_{\alpha_{ij}^{(l)}} \\ &= s_{\frac{T_{ij}^{(1)}}{\sum_{p=1}^{l} T_{ij}^{(p)}}\alpha_{ij}^{(1)} + \frac{T_{ij}^{(2)}}{\sum_{p=1}^{l} T_{ij}^{(p)}}\alpha_{ij}^{(2)} + \cdots + \frac{T_{ij}^{(l)}}{\sum_{p=1}^{l} T_{ij}^{(p)}}\alpha_{ij}^{(l)}}\end{aligned} \tag{12}$$

or the LPWG operator:

$$s_{\alpha_{ij}} = \mathrm{LPWG}\left(s_{\alpha_{ij}^{(1)}}, s_{\alpha_{ij}^{(2)}}, \ldots, s_{\alpha_{ij}^{(l)}}\right) = s_{\alpha_{ij}^{(1)}}^{\frac{T_{ij}^{(1)}}{\sum_{p=1}^{l} T_{ij}^{(p)}}} \otimes s_{\alpha_{ij}^{(2)}}^{\frac{T_{ij}^{(2)}}{\sum_{p=1}^{l} T_{ij}^{(p)}}} \otimes \cdots \otimes s_{\alpha_{ij}^{(l)}}^{\frac{T_{ij}^{(l)}}{\sum_{p=1}^{l} T_{ij}^{(p)}}} = s_{\left(\alpha_{ij}^{(1)}\right)^{\frac{T_{ij}^{(1)}}{\sum_{p=1}^{l} T_{ij}^{(p)}}} \left(\alpha_{ij}^{(2)}\right)^{\frac{T_{ij}^{(2)}}{\sum_{p=1}^{l} T_{ij}^{(p)}}} \cdots \left(\alpha_{ij}^{(l)}\right)^{\frac{T_{ij}^{(l)}}{\sum_{p=1}^{l} T_{ij}^{(p)}}}} \quad (13)$$

to aggregate all the individual linguistic decision matrices $A^{(k)} = \left(s_{\alpha_{ij}^{(k)}}\right)_{m\times n}$ $(k = 1, 2, \cdots, l)$ into the collective linguistic decision matrix $A = \left(s_{\alpha_{ij}}\right)_{m\times n}$.

Step 3: Calculate the matrix $T = \left(T_{ij}\right)_{m\times n}$ based on following equations:

$$T_{ij} = \prod_{k=1}^{j-1}\left(\frac{\alpha_{ik}}{g}\right) \left(i = 1, 2, \cdots, m,\ j = 2, \cdots, n\right), \quad (14)$$

$$T_{i1} = 1 \left(i = 1, 2, \cdots, m\right). \quad (15)$$

Step 4: Utilize the LPWA operator:

$$s_{\alpha_i} = \mathrm{LPWA}\left(s_{\alpha_{i1}}, s_{\alpha_{i2}}, \ldots, s_{\alpha_{in}}\right) = \frac{T_{i1}}{\sum_{j=1}^{n} T_{ij}} s_{\alpha_{i1}} \oplus \frac{T_{i2}}{\sum_{j=1}^{n} T_{ij}} s_{\alpha_{i2}} \oplus \cdots \oplus \frac{T_{in}}{\sum_{j=1}^{n} T_{ij}} s_{\alpha_{in}} = s_{\frac{T_{i1}}{\sum_{j=1}^{n} T_{ij}}\alpha_{i1} + \frac{T_{i2}}{\sum_{j=1}^{n} T_{ij}}\alpha_{i2} + \cdots + \frac{T_{in}}{\sum_{j=1}^{n} T_{ij}}\alpha_{in}} \quad (16)$$

or the LPWG operator:

$$s_{\alpha_i} = \mathrm{LPWG}\left(s_{\alpha_{i1}}, s_{\alpha_{i2}}, \ldots, s_{\alpha_{in}}\right) = s_{\alpha_{i1}}^{\frac{T_{i1}}{\sum_{j=1}^{n} T_{ij}}} \otimes s_{\alpha_{i2}}^{\frac{T_{i2}}{\sum_{j=1}^{n} T_{ij}}} \otimes \cdots \otimes s_{\alpha_{in}}^{\frac{T_{in}}{\sum_{j=1}^{n} T_{ij}}} = s_{\alpha_{i1}^{\frac{T_{i1}}{\sum_{j=1}^{n} T_{ij}}} \alpha_{i2}^{\frac{T_{i2}}{\sum_{j=1}^{n} T_{ij}}} \cdots \alpha_{in}^{\frac{T_{in}}{\sum_{j=1}^{n} T_{ij}}}} \quad (17)$$

to derive the collective overall preference value s_{α_i} of the alternative x_i.

Step 5: Rank the collective overall preference values s_{α_i} $(i = 1, 2, \cdots, m)$ in descending order.

Step 6: Rank all the alternatives x_i $(i = 1, 2, \cdots, m)$ and select the best one(s) in accordance with the collective overall preference values s_{α_i} $(i = 1, 2, \cdots, m)$.

Step 7: End.

3.4 An Illustrative Example

In this subsection, we use a concrete example (adapted from [61]) to illustrate the application of our method.

Example 3.1 [61]. In order to strengthen academic education, promote the building of teaching body, the school of management in a Chinese university wants to introduce oversea outstanding teachers. This introduction has been raised great attention from the school, university president d_1, dean of management school d_2, and human resource officer d_3 sets up the panel of decision makers which will take the whole responsibility for this introduction. They made strict evaluation for 5 candidates x_i $(i = 1, 2, 3, 4, 5)$ from the following four aspects: (1) morality c_1; (2) research capability c_2; (3) teaching skill c_3; and (4) education background c_4. University president has the absolute priority for decision making, dean of the management school comes next. That is, there is a prioritization between three decision makers expressed by the linear ordering $d_1 \succ d_2 \succ d_3$. In addition, this introduction will be in strict accordance with the principle of combine ability with political integrity. In three decision makers' opinion, there exists the prioritization relationship among these attributes, for example, the morality of the candidate is the most important, but the education background of the candidate is not so important comparing with other attributes. Therefore, the prioritization relationship can be denoted by: $c_1 \succ c_2 \succ c_3 \succ c_4$. Suppose that five candidates x_i $(i = 1, 2, 3, 4, 5)$ are to be evaluated using the linguistic term set

$$S = \left\{\begin{array}{l} s_0 = \text{extremely poor},\ s_1 = \text{very poor},\ s_2 = \text{poor},\ s_3 = \text{slightly poor},\ s_4 = \text{fair}, \\ s_5 = \text{slightly good},\ s_6 = \text{good},\ s_7 = \text{very good},\ s_8 = \text{extremely good} \end{array}\right\}$$

by three decision makers d_k $(k = 1, 2, 3)$ under the above four attributes c_j $(j = 1, 2, 3, 4)$, and three linguistic decision matrices $A^{(k)} = \left(s_{\alpha_{ij}^{(k)}}\right)_{5\times 4}$ $(k = 1, 2, 3)$ are listed in Tables 1-3, respectively.

Table 1. Decision matrix $A^{(1)}$ provided by d_1

	c_1	c_2	c_3	c_4
x_1	s_6	s_7	s_2	s_5
x_2	s_1	s_4	s_7	s_8
x_3	s_2	s_3	s_3	s_5
x_4	s_8	s_6	s_3	s_6
x_5	s_6	s_5	s_2	s_2

Table 2. Decision matrix $A^{(2)}$ provided by d_2

	c_1	c_2	c_3	c_4
x_1	s_6	s_2	s_3	s_3
x_2	s_7	s_4	s_7	s_8
x_3	s_4	s_6	s_6	s_5
x_4	s_8	s_6	s_3	s_5
x_5	s_3	s_1	s_1	s_2

Table 3. Decision matrix $A^{(3)}$ provided by d_3

	c_1	c_2	c_3	c_4
x_1	s_2	s_1	s_2	s_8
x_2	s_7	s_8	s_7	s_8
x_3	s_5	s_6	s_3	s_4
x_4	s_5	s_6	s_4	s_8
x_5	s_5	s_7	s_1	s_4

Step 1: We first utilize Eqs. (10) and (11) to calculate the matrices $T^{(1)}$, $T^{(2)}$, and $T^{(3)}$ as follows:

$$T^{(1)}=\begin{pmatrix} 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \end{pmatrix},$$

$$T^{(2)}=\begin{pmatrix} 0.7500 & 0.8750 & 0.2500 & 0.6250 \\ 0.1250 & 0.5000 & 0.8750 & 1.0000 \\ 0.2500 & 0.3750 & 0.3750 & 0.6250 \\ 1.0000 & 0.7500 & 0.3750 & 0.7500 \\ 0.7500 & 0.6250 & 0.2500 & 0.2500 \end{pmatrix},$$

$$T^{(3)}=\begin{pmatrix} 0.5625 & 0.2188 & 0.0938 & 0.2344 \\ 0.1094 & 0.2500 & 0.7656 & 1.0000 \\ 0.1250 & 0.2813 & 0.2813 & 0.3906 \\ 1.0000 & 0.5625 & 0.1406 & 0.4688 \\ 0.2813 & 0.0781 & 0.0313 & 0.0625 \end{pmatrix}.$$

Step 2: Utilize the LPWA operator (Eq. (12)) to aggregate all the individual linguistic decision matrices $A^{(k)}=\left(s_{\alpha_{ij}^{(k)}}\right)_{5\times 4}$ $(k=1,2,3)$ into the collective linguistic decision matrix $A=\left(s_{\alpha_{ij}}\right)_{5\times 4}$ (see Table 4).

Table 4. The collective decision matrix A

	c_1	c_2	c_3	c_4
x_1	$s_{5.0270}$	$s_{4.2836}$	$s_{2.1860}$	$s_{4.7059}$
x_2	$s_{2.1392}$	$s_{4.5714}$	$s_{7.0000}$	$s_{8.0000}$
x_3	$s_{2.6364}$	$s_{4.1887}$	$s_{3.6792}$	$s_{4.8062}$
x_4	$s_{7.0000}$	$s_{6.0000}$	$s_{3.0928}$	$s_{6.0845}$
x_5	$s_{4.7538}$	$s_{3.6239}$	$s_{1.7805}$	$s_{2.0952}$

Step 3: Calculate the matrix $T=\left(T_{ij}\right)_{5\times 4}$ based on Eqs. (14) and (15):

$$T=\begin{pmatrix} 1 & 0.6284 & 0.3365 & 0.0919 \\ 1 & 0.2674 & 0.1528 & 0.1337 \\ 1 & 0.3295 & 0.1725 & 0.0794 \\ 1 & 0.8750 & 0.6563 & 0.2537 \\ 1 & 0.5942 & 0.2692 & 0.0599 \end{pmatrix}.$$

Step 4: Utilize the LPWA operator (Eq. (16)) to aggregate all the preference values $s_{\alpha_{ij}}$ $(i=1,2,3,4,5)$ in the ith line of A, and derive the collective overall preference value s_{α_i} of the alternative x_i.

$s_{\alpha_1}=s_{4.3208}$, $s_{\alpha_2}=s_{3.5400}$, $s_{\alpha_3}=s_{3.1825}$, $s_{\alpha_4}=s_{5.6817}$, $s_{\alpha_5}=s_{3.9058}$.

Step 5: Rank the collective overall preference values s_{α_i} $(i=1,2,3,4,5)$ in descending order:

$$s_{\alpha_4}>s_{\alpha_1}>s_{\alpha_5}>s_{\alpha_2}>s_{\alpha_3}.$$

Step 6: Because $s_{\alpha_4}>s_{\alpha_1}>s_{\alpha_5}>s_{\alpha_2}>s_{\alpha_3}$, we have $x_4 \succ x_1 \succ x_5 \succ x_2 \succ x_3$. Therefore, the best candidate is x_4.

If we deal with Example 3.1 using the LPWG operator instead of the LPWA operator, then the main steps are shown as follows:

Step 1': See Step 1.

Step 2': Utilize the LPWG operator (Eq. (13)) to aggregate all the individual linguistic decision matrices $A^{(k)} = \left(s_{\alpha_{ij}^{(k)}}\right)_{5\times4}$ $(k = 1,2,3)$ into the collective linguistic decision matrix $A' = \left(s'_{\alpha_{ij}}\right)_{5\times4}$ (see Table 5).

Table 5. The collective decision matrix A'

	c_1	c_2	c_3	c_4
x_1	$s_{4.5930}$	$s_{3.3840}$	$s_{2.1567}$	$s_{4.4682}$
x_2	$s_{1.4470}$	$s_{4.4164}$	$s_{7.0000}$	$s_{8.0000}$
x_3	$s_{2.4657}$	$s_{3.9482}$	$s_{3.5098}$	$s_{4.7884}$
x_4	$s_{6.8399}$	$s_{6.0000}$	$s_{3.0812}$	$s_{5.9949}$
x_5	$s_{4.5294}$	$s_{2.8130}$	$s_{1.7177}$	$s_{2.0671}$

Step 3': Calculate the matrix $T' = \left(T'_{ij}\right)_{5\times4}$ based on Eqs. (14) and (15):

$$T' = \begin{pmatrix} 1 & 0.5741 & 0.2429 & 0.0655 \\ 1 & 0.1809 & 0.0998 & 0.0874 \\ 1 & 0.3082 & 0.1521 & 0.0667 \\ 1 & 0.8550 & 0.6412 & 0.2470 \\ 1 & 0.5662 & 0.1991 & 0.0427 \end{pmatrix}$$

Step 4': Utilize the LPWG operator (Eq. (17)) to aggregate all the preference values $s'_{\alpha_{ij}}$ $(i = 1,2,3,4,5)$ in the ith line of A', and derive the collective overall preference value s'_{α_i} of the alternative x_i.

$$s'_{\alpha_1} = s_{3.7920},\ s'_{\alpha_2} = s_{2.0986},\ s'_{\alpha_3} = s_{2.8912},\ s'_{\alpha_4} = s_{5.3852},\ s'_{\alpha_5} = s_{3.4422}.$$

Step 5': Rank the collective overall preference values s'_{α_i} $(i = 1,2,3,4,5)$ in descending order:

$$s'_{\alpha_4} > s'_{\alpha_1} > s'_{\alpha_5} > s'_{\alpha_3} > s'_{\alpha_2}.$$

Step 6': Because $s'_{\alpha_4} > s'_{\alpha_1} > s'_{\alpha_5} > s'_{\alpha_3} > s'_{\alpha_2}$, we have $x_4 \succ x_1 \succ x_5 \succ x_3 \succ x_2$. Therefore, the best candidate is x_4.

4. CONCLUSIONS

In some multiple attribute group decision making problems with linguistic information or uncertain linguistic information, there may exist a prioritization relationship over the attributes and decision makers. To deal with such cases, we in this paper develop some linguistic prioritized aggregation operators in which there exists a prioritization relationship between the arguments, such as the linguistic prioritized weighted average (LPWA) operator and the linguistic prioritized weighted geometric (LPWG) operator. We investigate some basic properties of these operators, such as idempotency, boundedness, and monotonicity. Furthermore, some multiple attribute group decision making methods based on the proposed operators are developed, and some concrete examples are given to verify our methods. It should be noted that the newly proposed approaches capture an important feature for decision making in a linguistic environment: there exists a prioritization relationship over the attributes and decision makers. Therefore, the new proposals are not only more reasonable but more efficient for some real-life applications of decision making in a linguistic environment.

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COMPETING INTERESTS

Author has declared that no competing interests exist.

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