British Journal of Mathematics & Computer Science
11(3): 1-6, 2015, Article no.BJMCS.20110
ISSN: 2231-0851

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Ranking Fuzzy Numbers by Using Dominance Based Approach

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Article Information

DOI: 10.9734/BJMCS/2015/20110
Editor(s):
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Complete Peer review History: http://sciencedomain.org/review-history/11272

Original Research Article

Received: 11 July 2015
Accepted: 19 August 2015
Published: 05 September 2015

Abstract

The most commonly used approach for ranking fuzzy numbers is ranking indices based on the centroids of fuzzy numbers. In this paper, we introduced a new ranking method which is based on multi-index such as relative dominance, priority weight and decision-makers preference. In addition to this we determined the feasibility and validity of this method by providing numerical examples.

Keywords: Fuzzy numbers; fuzzy relation; dominance-based approach; relative dominance; priority weight.

1 Introduction

Ranking fuzzy numbers play a very vital role in decision-making, optimization, data analysis and other applications. Therefore, various ranking approaches [1-5] have been proposed and investigated, among the ranking approaches, the centroid methods [4] are commonly used approach to rank fuzzy numbers. In addition to this, numerous other ranking techniques have been proposed by using the centroid concept [4].

This paper considers the relative importance of the centroid approach. Firstly, we define the relative dominance and priority weight. Our approach for ranking fuzzy numbers is based on the defined relative dominance, priority weight and the preference.

For the sake of clarity, the related concepts of fuzzy theories are presented in Section 2. And also, we discussed new approach for ranking fuzzy numbers based on centroid index, while, we list some valuable properties in Section 3. In Section 4, we have given an illustrative example. Finally, we summarized the main results and draws conclusion of this research in Section 5.

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2 Preliminaries

We review some basic definitions of fuzzy sets in this sections.

Definition 2.1.

A fuzzy number A is a subset of the real line R with membership function.

$$\mu_A(x) = \begin{cases} \mu_A^L(x) \text{if } a \leq x < b \\ w \quad \text{if } b \leq x \leq c \\ \mu_A^R(x) \text{if } c < x \leq d \\ 0 \quad \text{otherwise} \end{cases}$$

where, $0 \leq w \leq 1$ is a constant, $\mu_A^L(x)$ and $\mu_A^R(x)$ are two strictly monotonically and continuous mappings from to the closed interval $[0, w]$. If $w = 1$ then A is a normal fuzzy number, if not it is said to be a non-normal fuzzy number. Otherwise,A is referred to as a trapezoidal fuzzy number and it's usually denoted by $A = (a, b, c, d; w)$. Further, when $b \equiv c$, the trapezoidal fuzzy number is reduced to a triangular fuzzy number denoted by $A = (a, b, d; w)$. Hence, triangular fuzzy numbers are special case of trapezoidal fuzzy numbers.

Since, $\mu_A^L(x)$ and $\mu_A^R(x)$ are both strict monotonically and continuous functions, their inverse functions exist and should be continuous and strictly monotonical. Let $g_A^L(y)$ and $g_A^R(y)$ be the inverse functions of $\mu_A^L(x)$ and $\mu_A^R(x)$.

Apparently, there should be a strict inequalities in the piecewise domains of the membership function, so that the piecewise domains do not overlap. Further, $\mu_A^L(x)$ and $\mu_A^R(x)$ should have strict monotonicity, making sure that inverse function exist.

Definition 2.2.

The height of fuzzy number A is defined as the following relation

$$h(A) = sup\{\mu(x) | x \in S(A)\},$$

where, $S(A)$ is the support set of A.

Definition 2.3.

The left deviation degree of fuzzy number A_i, $i = 1,2, \cdots, n$ is defined in the following way.

$$S_i^L = \int_0^{w_i} g_{A_i}^L(y) dy,$$

where, $g_{A_i}^L(y)$ is the inverse function of $\mu_A^L(x)$ and $w_i = \min\{h(A_i) | i = 1,2, \cdots, n\}$.

Definition 2.4.

The priority weight of fuzzy numbers A_i, $i = 1,2, \cdots, n$ is defined as

$$\alpha_i = \frac{S_i^L}{\sum_{i=1}^n S_i^L},$$

where, S_i^L is the left deviation degree of A_i and $\sum_{i=1}^n S_i^L = 0$.

Definition 2.5.

The centroid point $(\bar{x}_i, \bar{y}_i)$ of fuzzy numbers A_i, $i = 1,2,\cdots,n$ is given as

$$\bar{x}_i = \frac{\int_{a_i}^{b_i} \mu_{A_i}^L(x)dx + \int_{b_i}^{c_i} wxdx + \int_{c_i}^{d_i} x\mu_{A_i}^R(x)dx}{\int_{a_i}^{b_i} \mu_{A_i}^L(x)dx + \int_{b_i}^{c_i} wdx + \int_{c_i}^{d_i} \mu_{A_i}^R(x)dx},$$

and

$$\bar{y}_i = \frac{\int_0^{w_i} y\left(g_{A_i}^R(y) - g_{A_i}^L(y)\right)dy}{\int_0^{w_i}\left(g_{A_i}^R(y) - g_{A_i}^L(y)\right)dy}.$$

where, $\mu_A^L(x)$ and $\mu_A^R(x)$ are the left and right membership function of fuzzy number A respectively, whereas, $g_A^L(y)$ and $g_A^R(y)$ are the inverse function of $\mu_A^L(x)$ and $\mu_A^R(x)$. $\int_{a_i}^{b_i} \mu_{A_i}^L(x)dx + \int_{b_i}^{c_i} wdx +$ *cidiμAiRxdx* and *0wigAiRy−gAiLydy* not equal to zero.

Definition 2.6.

The dominance of fuzzy numbers A_i, with respect to A_j is given by

$$d_{ij} = \begin{cases} \dfrac{\bar{x}_i}{\bar{x}_j} \text{ if } i \neq j \\ 1 \quad\quad \text{if } i = j \end{cases}$$

By the aforementioned definition, for a group of fuzzy numbers A_i, $i = 1,2,\cdots,n$, we can get it's the dominance matrix

$$D = \begin{bmatrix} d_{11} & \cdots & d_{1n} \\ \vdots & \ddots & \vdots \\ d_{n1} & \cdots & d_{nn} \end{bmatrix}$$

where, $d_{ij}d_{ji} = 1$, and $d_{ii} = 1$.

3 Ranking Method of Fuzzy Numbers Based on Centroid Index

The ranking index for fuzzy numbers A_i, $i = 1,2,\cdots,n$ is given by

$$R_i = \lambda \sum_{j=1}^{n} \alpha_i d_{ij} + (1-\lambda)\bar{y}_i$$

where, $\lambda \in [0.5,1]$ is the performance coefficient, which is determined by decision makers (generally $\lambda = 0.5$).

For any two fuzzy numbers A_i and A_j, the new ranking order is determined based on the following rules.

1. $A_i \succ A_j$, if and only if $R_i > R_j$, and
2. $A_i \sim A_j$, if and only if $R_i = R_j$.

Assume that there are n different fuzzy numbers $A_1, A_2, \cdots, A_n$. S is the support set of these fuzzy numbers. Note that this is a relative ranking measure, meaning that if two different classes of fuzzy numbers both consist fuzzy number A, then the ranking indices of A would be different, depending on the other fuzzy numbers in the classes.

3.1 Properties

Let A_i, A_j and A_k be any three fuzzy numbers, $i \neq j \neq k$ and $1 \leq i, j, k \leq n$.

1) If $A_i \succ A_j$ and $A_j \succ A_k$, then $A_i \succ A_k$.
2) If in f$\left(S(A_i)\right)$ ≥sup $\left(S\left(A_j\right)\right)$, then $A_i \succ A_j$
3) If $A_i \succ A_j$, then $A_i^{'} \succ A_j^{'}$, where, $A_i^{'} = (a_i + r, b_i + r, c_i + r, d_i + r, w)$ and $A_j^{'} = \left(a_j + r, b_j + r, cj+r, dj+r, w\right.$ are defined on $\mathcal{S}$ and r is a real number.
4) If $A_i \geq A_j$ and $A_i \leq A_j$, then $A_i \sim A_j$.
5) If $A_i \succ A_j (i \neq j,\ j = 1,2, \cdots, n\ )$, when we add one more fuzzy number A_{n+1}, $A_i \succ A_j$ is still true.

4 Illustrative Example

We used following numerical example in order to illustrate the approach of ranking fuzzy numbers. Let A_1 and A_2 be two "triangular" fuzzy numbers such that $A_1 = (1,2,2,3; 1)$ and $A_2 = \left(9,10,10,11; \frac{1}{10}\right)$

$$\mu_{A_1}(x) = \begin{cases} x - 1 & \text{if} \quad 1 \leq x < 2 \\ 1 & \text{if} \quad x = 2 \\ -x + 3 & \text{if} \quad 2 < x \leq 3 \\ 0 \text{otherwise} & \end{cases}$$

and

$$\mu_{A_2}(x) = \begin{cases} \frac{1}{10}x - \frac{9}{10} \text{if } 9 \leq x < 10 \\ \frac{1}{10} \text{if} \quad x = 10 \\ -\frac{1}{10}x + \frac{11}{10} \text{if} \quad 10 < x \leq 11 \\ 0 \text{otherwise} \end{cases}$$

Also, $g_{A_1}^L(y) = y + 1$ and $g_{A_1}^R(y) = -y + 3$ on [0,1) on the other hand, $g_{A_2}^L(y) = 10y + 9$ and $g_{A_2}^R(y) = -10y + 11$ on $[0, \frac{1}{10})$.

Step 1:

Now, $w_i = \min\left\{1, \frac{1}{10}\right\} = \frac{1}{10}$, we have

$$S_1^L = \int_0^{\frac{1}{10}} (y + 1) dy = \frac{21}{200}$$

and

$$S_2^L = \int_0^{\frac{1}{10}} (10y+9)dy = \frac{19}{20}$$

Step 2:

$$\alpha_1 = \frac{\frac{21}{200}}{\frac{21}{200}+\frac{19}{20}} = \frac{21}{211}$$

and

$$\alpha_2 = \frac{\frac{19}{20}}{\frac{21}{200}+\frac{19}{20}} = \frac{190}{211}$$

Step 3:

$$\bar{x}_1 = \frac{\int_1^2 x(x-1)dx + \int_2^2 \frac{1}{10} x dx + \int_2^3 x(-x+3)dx}{\int_1^2 (x-1)dx + \int_2^2 \frac{1}{10} dx + \int_2^3 (-x+3)dx} = 2,$$

and

$$\bar{y}_1 = \frac{\int_0^{\frac{1}{10}} y((-y+3)-(y+1))dy}{\int_0^{\frac{1}{10}}((-y+3)-(y+1))dy} = \frac{14}{285}.$$

The next one is,

$$\bar{x}_2 = \frac{\int_9^{10} x\left(\frac{x}{10}-\frac{9}{10}\right)dx + \int_{10}^{10} \frac{1}{10} x dx + \int_{10}^{11} x\left(-\frac{1}{10}x+\frac{11}{10}\right)dx}{\int_9^{10}\left(\frac{x}{10}-\frac{9}{10}\right)dx + \int_{10}^{10} \frac{1}{10} dx + \int_{10}^{11}\left(-\frac{1}{10}x+\frac{11}{10}\right)dx} = 10,$$

and

$$\bar{y}_i = \frac{\int_0^{\frac{1}{10}} y((-10y+11)-(10y+9))dy}{\int_0^{\frac{1}{10}}((-10y+11)-(10y+9))dy} = \frac{1}{30}.$$

Step 4:

$$d_{12} = \frac{2}{10} = \frac{1}{5},\ d_{21} = \frac{1}{d_{12}} = 5, \text{ and } d_{11} = d_{22} = 1.$$

Step 5:

Taking $\lambda = 0.5$, we have

$$R_1 = \frac{1}{2}\sum_{j=1}^{2} \alpha_1 d_{1j} + \left(1-\frac{1}{2}\right)\bar{y}_1$$

$$\frac{1}{2}\left(\frac{21}{211}\times 1+\frac{21}{211}\times\frac{2}{10}\right)+\left(1-\frac{1}{2}\right)\times\frac{14}{285}=\frac{5068}{60135}=0.08427$$

and

$$R_2=\frac{1}{2}\sum_{j=1}^{2}\alpha_1 d_{2j}+\left(1-\frac{1}{2}\right)\bar{y}_2$$

$$\frac{1}{2}\left(\frac{190}{211}\times 5+\frac{190}{211}\times 1\right)+\left(1-\frac{1}{2}\right)\times\frac{1}{30}=\frac{570}{211}+\frac{1}{60}=2.7180$$

So, the ranking order of fuzzy numbers is $A_2 \succ A_1$

5 Conclusion

In this paper, we defined relative dominance and priority weight then we proposed a new ranking approach based on centroid index. Further, we demonstrated our method by using an illustrative example and we found this method is simple and operates easily.

Competing Interests

Author has declared that no competing interests exist.

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