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Stochastic Analysis of Two Identical Unit Warm Standby System with Varying Demand of Production

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Abstract

This paper study the two unit warm stand by system in which the demand of items increases arbitrarily for some random amount of duration. Whenever demands of items to which the machines are producing is heavy the standby unit also starts operation and when the demand becomes Normal, the standby unit which is in operation comes into standby mode. Failure of the standby unit remains undetected therefore the standby unit is inspected at random intervals of time. The failure can also be detected at the time of need of standby unit to become operative. If the standby unit is found to be failed in the inspection then it is sent for repair immediately. Failure time distribution for both operative and standby units are assumed to be negative exponential. Regenerative point techniques with markov renewal process is used to obtain various reliability characteristics of system. Repair time distribution of units failed during operation and standby position are same and assumed to be general.

Keywords: Reliability; repair time; transition probability; regenerative points; mean sojourn time; MTSF; availability; markov renewal process.

MSC 2010 No.: Primary: 90B25, 62N05, 68M15 Secondary: 60K10, 60K20, 60K15.

1 Introduction

Several authors including [1-5] engaged in the field of reliability have analysed various engineering systems by using different sets of assumptions like fault detection, inspection, preventive maintenance, critical

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human error, etc. But in the real practical situations we can observe that some of the engineering systems operates accordingly to the demand of items which the machines are producing [1].

In [6], reliability characteristic of cold-standby redundant system was introduced. In [7], reliability modeling of 2-out-of-3 redundant system is introduced subject to degradation after repair. In [8], human error and common-cause failure modelling was established for a two-unit multiple system. In [9], stochastic analysis of a repairable system with three units and two repair facilities was introduced. In [10], some reliability parameters of a three state repairable system with environmental failure were evaluated.

Keeping the above view, we analysed a two unit warm standby system in which the demand of items increases arbitrarily for some random amount of duration. Whenever the demand increases the standby unit becomes operative provided both machines are alive [3]. After each repair the unit goes for installation which takes random amount of time to complete.

Using regenerative point technique with Markov renewal process, the following reliability characteristics of interest are obtained [2].

(1) Transition and steady state transition probabilities
(2) Mean Sojourn times in various states
(3) Mean time to system failure (MTSF)
(4) Point wise and Steady state availability of the system
(5) Expected Busy period of the repairman
(6) Expected number of visits by the repairman

2 Model Description and Assumptions

(1) The system consists of two identical units of machines. Initially, one unit is operative and the other is kept as warm standby.
(2) Whenever demands of items to which the machines are producing is heavy the standby unit also starts operation and when the demand becomes Normal, the standby unit which is in operation comes into standby mode.
(3) Failure of the standby unit remains undetected therefore the standby unit is inspected at random intervals of time. The failure can also be detected at the time of need of standby unit to become operative. If the standby unit is found to be failed in the inspection then it is sent for repair immediately.
(4) After each repair the unit goes for installation before starting its operation.
(5) A single repair facility with discipline FCFS is available for repair and inspection but installation of unit gets priority over repair.
(6) Failure time distribution for both operative and standby units are assumed to be negative exponential. Also the distributions for variations in demand from "normal to heavy" and "heavy to normal" and for inspection of standby unit are negative exponential while the distribution of repair and installation time are assumed to be general.
(7) Repair time distribution of units failed during operation and standby position are same and assumed to be general.

3 Notation and Symbols

N_0	:	Normal unit kept as operative
N_s	:	Normal unit kept as warm standby
F_r	:	Failed unit under repair
F_{wr}	:	Failed unit waiting for repair
F_R	:	Repair of failed unit continued from earlier state

F_u	:	Unit failed during standby position with undetected failure
F_i	:	Repaired unit under installation before starting its operation
F_l	:	Installation of repaired unit continued from earlier state
α	:	Constant failure rate of the operative unit
β	:	Constant failure rate of the warm standby unit
θ	:	Constant rate for increasing of demand from normal to heavy
η	:	Constant rate for decreasing of demand from heavy to normal
δ	:	Constant rate of inspection of standby unit
f(.), F(.)	:	pdf and cdf of time to repair a failed unit
g(.), G(.)	:	pdf and cdf of time to complete installation of the repaired unit
*	:	Symbol for Laplace Transformation
~	:	Symbol for Laplace Stieltjes Transform
©	:	Symbol for Ordinary Convolution
$	:	Symbol for Stieltjes Convolution

Using the above notation and symbols the possible states of the system are

<u>Up States</u>

$S_0 \equiv (N_0, N_s)$ $S_1 \equiv (N_0, N_0)$ $S_2 \equiv (N_o, F_r)$ $S_3 \equiv (N_0, F_i)$ $S_4 \equiv (N_0, F_u)$

<u>Down States</u>

$S_5 \equiv (F_r, F_{wr})$ $S_6 \equiv (F_{wr}, F_i)$ $S_7 \equiv (F_l, F_{wr})$ $S_8 \equiv (F_R, F_{wr})$

The transitions between the various states are shown in Fig. 1.

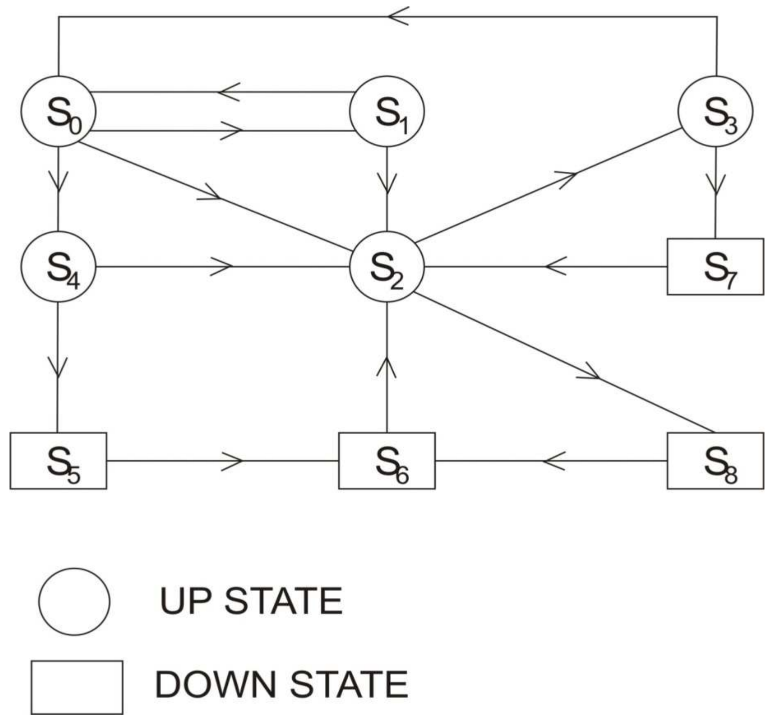

Fig. 1. State transition diagram

4 Transition Probabilities

Let T_o (=0), T_1,T_2,.... be the epochs at which the system enters the states $S_i \in$ E. Let X_n denotes the state entered at epoch T_{n+1} i.e. just after the transition of T_n. Then $\{T_n, X_n\}$ constitutes a Markov-renewal process with state space E and

$$Q_{ik}(t) = Pr[X_{n+1} = S_k, T_{n+1} - T_n \leq t \mid X_n = S_i] \tag{1}$$

is semi Markov-Kernal over E. The stochastic matrix of the embedded Markov chain is

$$P = P_{ik} = \lim_{t\to\infty} Q_{ik}(t) = Q(\infty) \tag{2}$$

By simple probabilistic consideration, the non-zero elements of $Q_{ik}(t)$ are:

$$Q_{01}(t) = {}_0\int^t \theta e^{-(\alpha+\beta+\theta)u} du$$

$$= \frac{\theta}{\alpha+\beta+\theta}[1-e^{-(\alpha+\beta+\theta)t}]$$

$$Q_{02}(t) = {}_0\int^t \alpha e^{-(\alpha+\beta+\theta)u} du$$

$$= \frac{\alpha}{\alpha+\beta+\theta}[1-e^{-(\alpha+\beta+\theta)t}]$$

$$Q_{04}(t) = {}_0\int^t \beta e^{-(\alpha+\beta+\theta)u} du$$

$$= \frac{\beta}{\alpha+\beta+\theta}[1-e^{-(\alpha+\beta+\theta)t}]$$

$$Q_{10}(t) = {}_0\int^t \eta e^{-(2\alpha+\eta)u} du$$

$$= \frac{\eta}{2\alpha+\eta}[1-e^{-(2\alpha+\eta)t}]$$

$$Q_{12}(t) = {}_0\int^t 2\alpha e^{-(2\alpha+\eta)u} du$$

$$= \frac{2\alpha}{2\alpha+\eta}[1-e^{-(2\alpha+\eta)t}]$$

$$Q_{23}(t) = {}_0\int^t e^{-\alpha u} f(u)du$$

$$Q_{28}(t) = {}_0\int^t \alpha e^{-\alpha u} \overline{F}(u)du$$

$$Q_{30}(t) = {}_0\int^t e^{-\alpha u} g(u)du$$

$$Q_{37}(t) = {}_0\int^t \alpha e^{-\alpha u} \overline{G}(u)du$$

$$Q_{42}(t) = {}_0\int^t \delta e^{-(\alpha+\delta)u} du$$

$$= \frac{\delta}{\alpha+\delta}[1 - e^{-(\alpha+\delta)t}]$$

$$Q_{45}(t) = {}_0\int^t \alpha e^{-(\alpha+\delta)u} du$$

$$= \frac{\alpha}{\alpha+\delta}[1 - e^{-(\alpha+\delta)t}]$$

$$Q_{56}(t) = {}_0\int^t f(u)du$$

$$Q_{62}(t) = {}_0\int^t g(u)du$$

$$Q^{(8)}{}_{26}(t) = {}_0\int^t (1 - e^{-\alpha v})dF(v)$$

$$Q^{(7)}{}_{32}(t) = {}_0\int^t (1 - e^{-\alpha v})dG(v) \tag{3-17}$$

Taking limit as $t \to \infty$, the steady state transition p_{ij} can be obtained from (3-17). Thus

$$p_{ik} = \lim_{t\to\infty} Q_{ik}(t) \tag{18}$$

$$p_{01} = \frac{\theta}{\alpha+\beta+\theta} \qquad p_{02} = \frac{\alpha}{\alpha+\beta+\theta}$$

$$p_{04} = \frac{\beta}{\alpha+\beta+\theta} \qquad p_{10} = \frac{\eta}{2\alpha+\eta}$$

$$p_{12} = \frac{2\alpha}{2\alpha+\eta} \qquad p_{23} = f^*(\alpha)$$

$$p_{28} = 1 - f^*(\alpha) \qquad p_{30} = g^*(\alpha)$$

$$p_{37} = 1 - g^*(\alpha) \qquad p_{42} = \frac{\delta}{\alpha + \delta}$$

$$p_{45} = \frac{\alpha}{\alpha + \delta} \qquad p_{56} = p_{62} = 1$$

$$p^{(8)}{}_{26} = 1 - f^*(\alpha) \qquad p^{(7)}{}_{32} = 1 - g^*(\alpha) \tag{19-32}$$

From the above probabilities the following relation can be verified as;

$p_{01} + p_{02} + p_{04} = 1$ $p_{10} + p_{12} = 1$

$p_{23} + p_{28} = p_{23} + p^{(8)}{}_{26} = 1$

$p_{30} + p_{37} = p_{30} + p^{(7)}{}_{32} = 1$

$p_{42} + p_{45} = 1$

$p_{56} = 1 = p_{62}$ (33-38)

5 Mean Sojourn Times

The mean time taken by the system in a particular state S_i before transiting to any other state is known as mean sojourn time and is defined as

$$\mu_i = {}_0\int^{\infty} P[T > t]\, dt \tag{39}$$

where T is the time of stay in state S_i by the system.

To calculate mean sojourn time μ_i in state S_i, we assume that so long as the system is in state S_i, it will not transit to any other state. Therefore;

$$\mu_0 = \frac{1}{\alpha + \beta + \theta}$$

$$\mu_1 = \frac{1}{2\alpha + \eta}$$

$$\mu_2 = \frac{1}{\alpha}[1 - f^*(\alpha)]$$

$$\mu_3 = \frac{1}{\alpha}[1 - g^*(\alpha)]$$

$$\mu_4 = \frac{1}{\alpha + \delta}$$

$$\mu_5 = {}_0\int^{\infty} \overline{F}(t)dt = {}_0\int^{\infty} t.f(t)dt = m_1$$

$$\mu_6 = {}_0\int^{\infty} \overline{G}(t)dt = {}_0\int^{\infty} t.g(t)dt = m_2 = \mu_7 \tag{40-46}$$

5.1 Contribution to Mean Sojourn Time

For the contribution to mean sojourn time in state $S_i \in E$ and non-regenerative state occurs, before transiting to $S_j \in E$ i.e.,

$$m_{ij} = -\int t.q_{ij}(t)dt = -q'*_{ij}(0) \tag{47}$$

Therefore,

$$m_{01} = \frac{\theta}{(\alpha + \beta + \theta)^2}$$

$$m_{02} = \frac{\alpha}{(\alpha + \beta + \theta)^2}$$

$$m_{04} = \frac{\beta}{(\alpha + \beta + \theta)^2}$$

$$m_{10} = \frac{\eta}{(2\alpha + \eta)^2}$$

$$m_{12} = \frac{2\alpha}{(2\alpha + \eta)^2}$$

$$m_{23} = {}_0\int^{\infty} t.e^{-\alpha t} f(t)dt$$

$$m_{28} = {}_0\int^{\infty} t.\alpha.e^{-\alpha t} \overline{F}(t)dt$$

$$m_{30} = {}_0\int^{\infty} t.e^{-\alpha t} g(t)dt$$

$$m_{37} = {}_0\int^{\infty} t.\alpha.e^{-\alpha t} \overline{G}(t)dt$$

$$m_{42} = \frac{\delta}{(\alpha+\delta)^2}$$

$$m_{45} = \frac{\alpha}{(\alpha+\delta)^2}$$

$$m_{56} = {}_0\int^{\infty} t.f(t)dt$$

$$m_{62} = {}_0\int^{\infty} t.g(t)dt$$

$$m^{(8)}{}_{26} = {}_0\int^{\infty} t.(1-e^{-\alpha t})f(t)dt$$

$$m^{(7)}{}_{32} = {}_0\int^{\infty} t.(1-e^{-\alpha t})g(t)dt \quad (48\text{-}62)$$

Hence using (48 – 62) the following relations can be verified as follows

$m_{01} + m_{02} + m_{04} = \mu_0$ $\quad$ $m_{10} + m_{12} = \mu_1$

$m_{23} + m_{28} = \mu_2$ $\quad$ $m_{23} + m^{(8)}{}_{26} = m_1$

$m_{30} + m_{37} = \mu_3$ $\quad$ $m_{30} + m^{(7)}{}_{32} = m_2$

$m_{42} + m_{45} = \mu_4$

$$m_{56} = {}_0\int^{\infty} \bar{F}(t)dt = {}_0\int^{\infty} t.f(t)dt = \mu_5$$

$$m_{62} = {}_0\int^{\infty} \bar{G}(t)dt = {}_0\int^{\infty} t.g(t)dt = \mu_6 \quad (63\text{-}71)$$

6 Mean Time to System Failure

To obtain the distribution function $\pi_i(t)$ of the time to system failure with starting state S_0.

$\pi_0(t) = Q_{01}(t)\ \$\pi_1(t) + Q_{02}(t)\ \$\pi_2(t) + Q_{04}(t)\ \$\pi_4(t)$

$\pi_1(t) = Q_{10}(t)\ \$\pi_0(t) + Q_{12}(t)\ \$\ \pi_2(t)$

$\pi_2(t) = Q_{23}(t)\$\pi_3(t) + Q_{28}(t)$

$\pi_3(t) = Q_{30}(t)\$\pi_0(t) + Q_{37}(t)$

$\pi_4(t) = Q_{42}(t)\$\pi_2(t) + Q_{45}(t)$ (72-76)

Taking Laplace Stieltjes transform of relations (72-76) we have

$$\tilde{\pi}_0(s) = \tilde{Q}_{01}(s).\tilde{\pi}_1(s) + \tilde{Q}_{02}(s).\tilde{\pi}_2(s) + \tilde{Q}_{04}(s).\tilde{\pi}_4(s)$$

$$\tilde{\pi}_1(s) = \tilde{Q}_{10}(s).\tilde{\pi}_0(s) + \tilde{Q}_{12}(s).\tilde{\pi}_2(s)$$

$$\tilde{\pi}_2(s) = \tilde{Q}_{23}(s).\tilde{\pi}_3(s) + \tilde{Q}_{28}(s)$$

$$\tilde{\pi}_3(s) = \tilde{Q}_{30}(s).\tilde{\pi}_0(s) + \tilde{Q}_{37}(s)$$

$$\tilde{\pi}_4(s) = \tilde{Q}_{42}(s).\tilde{\pi}_2(s) + \tilde{Q}_{45}(s) \qquad (80\text{-}84)$$

and solving it for $\tilde{\pi}_0(s)$ by omitting the argument 's' for brevity,

$$\tilde{\pi}_0(s) = N_1(s) / D_1(s) \qquad (85)$$

where

$$N_1(s) = \tilde{Q}_{01}\tilde{Q}_{12}\tilde{Q}_{23}\tilde{Q}_{37} + \tilde{Q}_{04}\tilde{Q}_{42}\tilde{Q}_{23} + \tilde{Q}_{01}\tilde{Q}_{12}\tilde{Q}_{28} + \tilde{Q}_{04}\tilde{Q}_{42}\tilde{Q}_{28}$$

$$+\tilde{Q}_{02}\tilde{Q}_{23}\tilde{Q}_{37} + \tilde{Q}_{30}\tilde{Q}_{02}\tilde{Q}_{28} + \tilde{Q}_{04}\tilde{Q}_{45} \qquad (86)$$

and

$$D_1(s) = 1 - \tilde{Q}_{01}\tilde{Q}_{10} - \tilde{Q}_{02}\tilde{Q}_{23}\tilde{Q}_{30} - \tilde{Q}_{01}\tilde{Q}_{12}\tilde{Q}_{23}\tilde{Q}_{30} \qquad (87)$$

Then

$$N_1(0) = \lim_{s\to 0} N_1(s)$$

$$= p_{01}p_{12}p_{23}p_{37} + p_{04}p_{42}p_{23} + p_{01}p_{12}p_{28} + p_{04}p_{42}p_{28} + p_{02}p_{23}p_{37} + p_{30}p_{02}p_{28} + p_{04}p_{45} \qquad (88)$$

and

$$D_1(0) = \lim_{s\to 0} D_1(s)$$

$$= 1 - p_{01}p_{10} - p_{02}p_{23}p_{30} - p_{01}p_{12}p_{23}p_{30} \qquad (89)$$

Here by further simplifications it can be seen that

$N_1(0) = D_1(0)$,

Therefore,

$\tilde{\pi}_0(0) = N_1(0) / D_1(0) = 1$

This implies that $\tilde{\pi}_0(t)$ is proper distribution function.

Now, to obtain the relevant coefficient of m_{ij} and $m^{(k)}_{ij}$ in $D'_1(0) – N'_1(0)$, we proceed as follows Coefficient of m_{ij}'s in $D'_1(0) - N'_1(0)$

Coefficient of m_{ij}'s in $D'_1(0) – N'_1(0)$

m_{ij}	Coefficient
m_{01}	1
m_{02}	1
m_{04}	1
m_{10}	p_{01}
m_{12}	p_{01}
m_{23}	$p_{01}p_{12} + p_{02}p_{30} + p_{04}p_{42}$
m_{28}	$p_{01}p_{12} + p_{02}p_{30} + p_{04}p_{42}$
m_{30}	$p_{23}(p_{01}p_{12} + p_{02})$
m_{37}	$p_{23}(p_{01}p_{12} + p_{02})$
m_{42}	$p_{28}p_{04}$
m_{45}	$p_{28}p_{04}$

Therefore,

$$D'_1(0) - N'_1(0) = (m_{01} + m_{02} + m_{04}) + (m_{10} + m_{12})p_{01}$$

$$+ (m_{23} + m_{28})(p_{01}p_{12} + p_{02}p_{30} + p_{04}p_{42})$$

$$+ (m_{30} + m_{37})p_{23}(p_{01}p_{12} + p_{02})$$

$$+ (m_{42} + m_{45})p_{28}p_{04}$$

$$= \mu_0 + \mu_1 p_{01} + m_1(p_{01}p_{12} + p_{30}p_{02} + p_{04}p_{42})$$

$$+ m_2(p_{01}p_{12}p_{23} + p_{02}p_{23}) + \mu_4 p_{28}p_{04} \qquad (90)$$

Therefore, mean time to system failure when the initial state is S_o, is

$$E(T) = -\frac{d}{dx}\pi_0(s)|_{s=0} = \frac{D'_1(0) - N'_1(0)}{D_1(0)} = N_1 / D_1 \qquad (91)$$

where N_1 and D_1 are same as in (90) and (89) respectively.

7 Availability Analysis

System availability is defined as

$A_i(t)$ = Pr[Starting from state S_i the system is available at epoch t without passing through any regenerative state] and $M_i(t)$ = Pr[Starting from up state S_i the system remains up till epoch t without passing through any regenerative up state]

Obtaining $A_i(t)$ by using elementary probability argument;

$$A_0(t) = M_0(t) + q_{01}(t)©A_1(t) + q_{02}(t)©A_2(t) + q_{04}(t) ©A_4(t)$$

$$A_1(t) = M_1(t) + q_{10}(t) © A_0(t) + q_{12}(t) © A_2(t)$$

$A_2(t) = M_2(t) + q_{23}(t) © A_3(t) + q^{(8)}_{26}(t) © A_6(t)$

$A_3(t) = M_3(t) + q_{30}(t) © A_0(t) + q^{(7)}_{32}(t) © A_2(t)$

$A_4(t) = M_4(t) + q_{42}(t) © A_2(t) + q_{45}(t) © A_5(t)$

$A_5(t) = q_{56}(t) © A_6(t)$

$A_6(t) = q_{62}(t) © A_2(t)$ (92-98)

Where

$M_0(t) = e^{-(\alpha+\beta+\theta)t}$ $M_1(t) = e^{-(2\alpha+\eta)t}$

$M_2(t) = e^{-\alpha t}\overline{F}(t)$ $M_3(t) = e^{-\alpha t}\overline{G}(t)$

$M_4(t) = e^{-(\delta+\alpha)t}$ (99-103)

Taking Laplace transform of above equation (92-98) we have

$A^*_0(s) = M^*_0(s) + q^*_{01}(s).A^*_1(s) + q^*_{02}(s).A^*_2(s) + q^*_{04}(s).A^*_4(s)$ $A^*_1(s)$

$= M^*_1(s) + q^*_{10}(s).A^*_0(s) + q^*_{12}(s).A^*_2(s)$ $A^*_2(s) = M^*_2(s)$

$+ q^*_{23}(s).A^*_3(s) + q^{*(8)}_{26}(s).A^*_6(s)$ $A^*_3(s) = M^*_3(s) + q^*_{30}(s).A^*_0(s)$

$+ q^{*(7)}_{32}(s).A^*_2(s)$ $A^*_4(s) = M^*_4(s) + q^*_{42}(s).A^*_2(s) + q^*_{45}(s).A^*_5(s)$

$A^*_5(s) = q^*_{56}(s).A^*_6(s)$ $A^*_6(s) = q^*_{62}(s).A^*_2(s)$ (104-110)

Now, solving for point wise availability $A^*_0(s)$, by omitting the arguments 's' for brevity, one gets

$$A^*_0(s) = \frac{N_2(s)}{D_2(s)} \quad (111)$$

Where

$N_2(s) = [(M^*_0 + q^*_{01}M_1 + q^*_{04}M^*_4).(1 - q^{*(7)}_{32}q^*_{23} - q^{*(8)}_{26}q^*_{62})$

$+ (M^*_2 + q^*_{23}M_3)(q^*_{01}q^*_{12} + q^*_{02} + q^*_{04}q^*_{42} + q^*_{04}q^*_{45}\ q^*_{56}q^*_{62})]$ (112)

and

$D_2(s) = [(1 - q^{*(7)}_{32}q^*_{23} - q^{*(8)}_{26}q^*_{62})(1 - q^*_{01}q^*_{10})$

$- q^*_{23}q^*_{30}(q^*_{01}q^*_{12} + q^*_{02} + q^*_{04}q^*_{42} + q^*_{04}q^*_{45}q^*_{56}q^*_{62})]$ (113)

Then

$D_2(0) = [(1 - p^{(7)}_{32}p_{23} - p^{(8)}_{26})(1 - p_{01}p_{10})$

$- p_{23}p_{30}(p_{01}p_{12} + p_{02} + p_{04}p_{42} + p_{04}p_{45})]$ (114)

Now, we collect the relevant coefficients of $m_{ij} = -q*'_{ij}(0)$ in $D'_2(0)$ as follows

Coefficient of m_{ij}'s in $D'_2(0)$

m_{ij}'s	Coefficients
m_{01}	$p_{23}p_{30}$
m_{02}	$p_{23}p_{30}$
m_{04}	$p_{23}p_{30}$
m_{10}	$p_{01}p_{23}p_{30}$
m_{12}	$p_{01}p_{23}p_{30}$
m_{23}	$1 - p_{01}p_{10}$
$m^{(8)}_{26}$	$1 - p_{01}p_{10}$
m_{30}	$p_{23}(1 - p_{01}p_{10})$
$m^{(7)}_{32}$	$p_{23}(1 - p_{01}p_{10})$
m_{42}	$p_{23}p_{30}p_{04}$
m_{45}	$p_{23}p_{30}p_{04}$
m_{56}	$p_{23}p_{30}p_{04}p_{45}$
m_{62}	$p_{23}p_{30}p_{04}p_{45}$

Also, we can have that

$$M^*_0(0) = \mu_0 \qquad M^*_1(0) = \mu_1$$

$$M^*_2(0) = \mu_2 \qquad M^*_3(0) = \mu_3$$

$$M^*_4(0) = \mu_4 \tag{115-119}$$

Thus

$$D_2 = D_2'(0) = [p_{23}p_{30}(\mu_0 + \mu_1 p_{01} + \mu_4 p_{04})$$

$$+ (1 - p_{01}p_{10})(m_1 + m_2 p_{23}) + (\mu_6 + \mu_5)\, p_{23}p_{30}p_{04}p_{45})] \tag{120}$$

And

$$N_2 = N_2(0) = [p_{23}p_{30}(\mu_0 + \mu_1 p_{01} + \mu_4 p_{04}) + (1 - p_{01}p_{10})(\mu_2 + p_{23}\mu_3)] \tag{121}$$

Therefore, the steady state functioning availability of the system is

$$A_0(\infty) = \lim_{t\to\infty} A_0(t) = \lim_{s\to\infty} s.\ A_0*(s) = N_2(0)/D'_2(0) = N_2/D_2 \tag{122}$$

where N_2 and D_2 are given in (121) and (120) respectively.

8 Busy Period Ananlysis

Let $B_i(t)$ be the probability that the system is under repair at time t, Thus the following recursive relations among $B_i(t)$'s can be obtained as ;

$$B_0(t) = q_{01}(t) © B_1(t) + q_{02}(t) © B_2(t) + q_{04}(t) © B_4(t)$$

$$B_1(t) = q_{10}(t) © B_0(t) + q_{12}(t) © B_2(t)$$

$B_2(t) = W_2(t) + q_{23}(t) © B_3(t) + q^{(8)}_{26}(t) © B_6(t)$

$B_3(t) = W_3(t) + q_{30}(t) © B_0(t) + q^{(7)}_{32}(t) © B_2(t)$

$B_4(t) = q_{42}(t) © B_2(t) + q_{45}(t) © B_5(t)$

$B_5(t) = W_5(t) + q_{56}(t) © B_6(t)$

$B_6(t) = W_6(t) + q_{62}(t) © B_2(t)$ (123-129)

where

$W_2(t) = \bar{F}(t)$ $W_3(t) = \bar{G}(t)$

$W_5(t) = \bar{F}(t)$ $W_6(t) = \bar{G}(t)$ (130-133)

Taking Laplace transform of the equations (123-129), we get

$B^*_0(s) = q^*_{01}(s).B^*_1(s) + q^*_{02}(s).B^*_2(s) + q^*_{04}(s).B^*_4(s)$

$B^*_1(s) = q^*_{10}(s).B^*_0(s) + q^*_{12}(s).B^*_{12}(s)$

$B^*_2(s) = W^*_2(s) + q^*_{23}(s) . B^*_3(s) + q^{*(8)}_{26}(s).B^*_6(s)$

$B^*_3(s) = W^*_3(s) + q^*_{30}(s) . B^*_0(s) + q^{*(7)}_{32}(s).B^*_2(s)$

$B^*_4(s) = q^*_{42}(s).B^*_2(s) + q^*_{45}(s).B^*_5(s)$

$B^*_5(s) = W^*_5(s) + q^*_{56}(s).B^*_6(s)$

$B^*_6(s) = W^*_6(s) + q^*_{62}(s).B^*_2(s)$ (134-140)

Solving the above equations (134-140) for $B^*_0(s)$, by omitting the argument 's' for brevity we get;

$B^*_0(s) = N_3(s) / D_3(s)$ (141)

Where

$$N_3(s) = [(W^*_2 + q^{*(8)}_{26}W^*_6 + q^*_{23}W^*_3)(q^*_{01}q^*_{12} + q^*_{02} + q^*_{04}q^*_{45}q^*_{56}q^*_{62}$$
$$+q^*_{04}q^*_{42}) + q^*_{04}q^*_{45}(W^*_5 + q^*_{56}W^*_6)(1 - q^{*(8)}_{26}q^*_{62} - q^*_{23}q^{*(7)}_{32})] \quad (142)$$

and $D_3(s)$ is same as $D_2(s)$ in (113).

In the steady state, the fraction of time for which the repair facility is busy in repair is given by

$$B_0 = \lim_{t\to\infty} B_0(t) = \lim_{s\to\infty} B^*(s) = N_3(0) / D'_3(0) = N_3/D_3 \quad (143)$$

where in terms of

$W^*_2(0) = W^*_5(0) = m_1$
$W^*_3(0) = W^*_6(0) = m_2$ (144 – 145)

$N_3(0) = N_3 = [(m_1 + m_2)(1 - p_{01}p_{10} - p_{23}p_{30}p_{04}p_{45})]$ (146)

and D_3 is same as D_2 in (120).

9 Expected Number of Visits by the Repair Facility

Let we define, $V_i(t)$ as the expected number of visits by the repair facility in (0,t] given that the system initially started from regenerative state S_i at t = 0. Then following recurrence relations among $V_i(t)$'s can be obtained as;

$$V_0(t) = Q_{01}(t)\$V_1(t) + Q_{02}(t)\$[1 + V_2(t)] + Q_{04}(t)\$V_4(t)$$

$$V_1(t) = Q_{10}(t)\$V_0(t) + Q_{12}(t)\$[1 + V_2(t)]$$

$$V_2(t) = Q_{23}(t)\$V_3(t) + Q^{(8)}{}_{26}(t)\$V_6(t)$$

$$V_3(t) = Q_{30}(t)\$V_0(t) + Q^{(7)}{}_{32}(t)\$V_2(t)$$

$$V_4(t) = Q_{42}(t)\$V_2(t) + Q_{45}(t)\$[1+V_5(t)]$$

$$V_5(t) = Q_{56}(t)\$V_6(t)$$

$$V_6(t) = Q_{62}(t)\$V_2(t) \quad (147\text{-}153)$$

Taking Laplace-Stieltjes transform of the above equations (147-153), we get

$$\tilde{V}_0(s) = \tilde{Q}_{01}(s)\tilde{V}_1(s)] + \tilde{Q}_{02}(s).[1 + \tilde{V}_2(s)] + \tilde{Q}_{04}(s)\tilde{V}_4(s)$$

$$\tilde{V}_1(s) = \tilde{Q}_{10}(s).\tilde{V}_0(s) + \tilde{Q}_{12}(s).[1 + \tilde{V}_2(s)]$$

$$\tilde{V}_2(s) = \tilde{Q}_{23}(s).\tilde{V}_3(s) + \tilde{Q}_{26}^{(8)}(s).\tilde{V}_6(s)$$

$$\tilde{V}_3(s) = \tilde{Q}_{30}(s).\tilde{V}_0(s) + \tilde{Q}_{32}^{(7)}(s).\tilde{V}_2(s)$$

$$\tilde{V}_4(s) = \tilde{Q}_{42}(s).\tilde{V}_2(s) + \tilde{Q}_{45}(s).[1 + \tilde{V}_5(s)]$$

$$\tilde{V}_5(s) = \tilde{Q}_{56}(s).\tilde{V}_6(s)$$

$$\tilde{V}_6(s) = \tilde{Q}_{62}(s).\tilde{V}_2(s) \quad (154\text{-}160)$$

And the solution of $\tilde{V}_0(s)$ may be expressed as by omitting the argument's' for brevity is

$$\tilde{V}_0(s) = N_4(s)/D_4(s) \quad (161)$$

where

$$N_4(s) = (\tilde{Q}_{02} + \tilde{Q}_{01}\tilde{Q}_{12} - \tilde{Q}_{01}\tilde{Q}_{12}\tilde{Q}_{26}^{(8)}\tilde{Q}_{62} - \tilde{Q}_{01}\tilde{Q}_{12}\tilde{Q}_{23}\tilde{Q}_{32}^{(7)} + \tilde{Q}_{02}\tilde{Q}_{26}^{(8)}\tilde{Q}_{62} - \tilde{Q}_{02}\tilde{Q}_{23}\tilde{Q}_{32}^{(7)})] \quad (162)$$

and $D_4(s)$ can be obtained as $D_2(s)$ in (113) by putting Q_{ij} in place of q^*_{ij}.

Now,

$$N_4 = N_4(0) = p_{23}p_{30}(p_{01}p_{12} + p_{02}) \quad (163)$$

Therefore, In steady state the number of visit per unit of time when the system starts after entrance into state S_0 is;

$$V_0 = \lim_{t\to\infty}[V_0(t)/t] = \lim_{s\to\infty} s\ \tilde{V}_0(s) = N_4/D_4 \quad (164)$$

where N_4 and $D_4(=D_2)$ are as in (163) and (120) respectively.

10 Conclusion

The objective of this paper was to formulate a methodology for analysis two identical unit warm stand by system subject to varying demand of production of items increase arbitrarily for some random amount of time and common cause failures and general repair rate. The problem of evaluation of various reliability performance measures as Transition and steady state transition probabilities, Mean Sojourn times in various states, Mean time to system failure (MTSF), Point wise and Steady state availability, Expected Busy period of the repairman and Expected number of visits by the repairman according to the system using regenerative point technique with Markov renewal process which is convenient for computation. The result obtained in this paper can be applied to similar other model.

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Competing Interests

Authors have declared that no competing interests exist.

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