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Elastic Bending Deformation of the Drill Strings in Channels of Curve Wells

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Abstract

The problem about identification of elastic bending deformation of a drill string in a curve wells based on the theory of flexible curved rods and the direct inverse problems of drill string bending in the channels of curvilinear bore-holes is stated. The problem is solved which determines the resistance forces and moments during performing ascending-descending operations in curvilinear bore-holes with trajectories of the second order curve shapes. The sensitivity of the resistance forces relative to geometric parameters of the bore-hole axial line trajectories is analyzed.

Keywords

Curvilinear Drilling, Elastic Bending, Curve Well, Circular Friction, Resistance Forces

1. Introduction

Currently the time of easy oil and natural gas is completed [1] [2]. In as much as the reserves of hydrocarbon fuels in easy-to-extract basins approach to completion, the deposits located at the depths of 10 km become to be promising. Now the old bore-hole drilling using vertical wells is being redeveloped by horizontal and inclined wells [3]. But the experience gained while drilling vertical wells is not useful for drilling horizontal ones, because mechanical behavior of a drill string (DS) with curvilinear axial line acquires a series of specificities leading to critical situations. At oil and gas extraction from super deep levels, the efficiency enhancement of bore-holes drilling is associated with solution of the problems of revealing the critical regimes of the drill column functioning and elaboration of measures for their preventing. Curved wells are becoming increasingly common throughout the world, as they penetrate the oil-bearing and gas-bearing strata along their laminated structure and, therefore, cover greater areas of fuel consumption.

With the use of curved holes, the total number of through wells decreases and their migratory flow rate is ten times greater than the migratory flow rate of vertical wells.

However, the practical implementation of the technology of drilling such wells is associated with the necessity of theoretical modelling of mechanical phenomena that occur in the construction of drilling equipment in order to prevent the critical and emergency operation modes. Meanwhile the greatest interest is focused on determination of contact and friction forces, as well as the full resistance force acting on the drill string (DS) in a curved borehole during its descent, ascent and operation.

However, in the structural mechanics of curved rods, the methods of theoretical modelling of these phenomena are not developed yet. This state of matter is associated with significant complexity of these processes caused by the great length of the drill string (DS) and conditions of contact interaction with the walls of curved holes.

Given that the current depth of the wells is 9 to 10 km, the range of their routing is over 12 km, more than USD 50 million worth, and every third hole is in emergency condition, so it can be noted that the problem of theoretical modelling of the tube boring columns bending mechanics in channels of deep curved holes in view of their contact interaction is very important. In this paper, for theoretical modelling of elastic bending deformation of the DS in a curved borehole the classical theory of flexible curved rods is used [4].

Many researches and papers deal with this issue, A. Abdul-Ameer, 2012, the unsupported deflection of drill strings which are subjected to increasing gravitational loading and distortion is discussed here, taking into account borehole depth and twisting. Also eccentric loading configurations which include the lateral reaction to mud flow pressure, compression, bending forces and twisting moments are taking into account too. Also in this study the bending-buckling deflections and twist angle dynamics following drill string drive motor voltage changes are discussed. At specified cutting velocities the principal stress level is identified [5].

V. I. Gulyayev, *et al.* (2013) discussed the computer simulation behaviour of drill strings in hyper deep vertical, inclined and horizontal wells. Stability and post-critical non-linear deforming of the drill strings are considered. It is found that all of these parameters are singularly perturbed from the mathematical point of view and because of this; such parameters are poorly amenable to theoretical analysis. The software used in this study of these phenomena is elaborated. Such elaborated software permits one to construct its trajectory securing the smallest values of resistance forces and to choose the least energy-consuming and safe regimes of drilling [6].

Nabil W. Musa, *et al.* (2015) discussed the problem of simulation of a drill bit whirling under conditions of its contact interaction with the bore-hole bottom rock plane. It was showed that the problem on rolling of a rigid ellipsoidal drill bit on a bore-hole bottom plane, the bit is supposed to be attached to the lower end of an elastic rotating drill string. In this study two mechanical models of the bit motions based on assumption of the possibilities of its pure rolling whirling and rolling with sliding are discussed. The paper discussed the analogy between the phenomena of nonholonomic motions of the Celtic stones and the ellipsoidal bits. Also techniques of solution of the equations of the bit whirling were proposed [7].

The critical buckling of drill strings in cylindrical cavities of inclined bore-holes was been discussed by Nabil W. Musa, *et al.* (2016) [8], it was stated the mathematical model for computer analysis of bifurcation buckling of a drill in cylindrical channel of an inclined bore-hole.

2. Statement of Problems about Elastic Bending of the Drill String in Curve Wells

The general formulations of direct and inverse problems of deformation of the curved rods were presented in [9] [10] [11]. The applied aspects of this problem were considered in [12] [13]. In this paper formulated the problems of bending of the boring columns in curved well channels, and the purpose to develop the methods of modelling of the elastic bending of the column in a curved borehole and the study of the type of impact of the outlines of the central line of the well on the value of static parameters of the deformed DS upon descent-ascent operations.

In practice, the DS descent and ascent are usually performed by simultaneous provision of two types of movement to the column, axial and rotational. Using such technology of the descent-ascent operations one can achieve a favourable redistribution of axial and circular friction contact forces between the DS surface and well wall, upon which the whole selected operation will be less energy-intensive and, and the axial force at the top of the DS performing this procedure will be reduced significantly. The problem of quasi-static modelling of the drill strings during the descent-ascent operations with concomitant rotation depends on the contour of its axial line.

If the axial line of the well is preset by the equalities

$$x = x(\vartheta), \quad y = 0, \quad z = z(\vartheta) \tag{1}$$

where ϑ is a certain independent variable measured in meters.

The curvature and torsion options of this curve can be presented as:

$$p = \frac{1}{R}\sin\chi, \quad q = \frac{1}{R}\cos\chi, \quad r = \frac{\mathrm{d}\chi}{D\mathrm{d}\vartheta}. \tag{2}$$

Here χ is the angle between the $\boldsymbol{n}$ unit vector and the u-axis (which is part of the moving right-handed co-ordinate system (u, v, w); R is the curvature radius; p, q, r are the curvatures of the bore-hole axis line. And D is defined as $D = \sqrt{(\mathrm{d}x/\mathrm{d}\vartheta)^2 + (\mathrm{d}y/\mathrm{d}\vartheta)^2 + (\mathrm{d}z/\mathrm{d}\vartheta)^2}$.

Let's assume that in the initial state the axial line of the well is straight and $p_0 = 0$, $q_0 = 0$, $r_0 = 0$. Then the problem of elastic bending deformation of the DS in a curved based on the theory of flexible curved rods will be determined by equations of equilibrium (3).

$$\begin{aligned}
&\frac{\mathrm{d}F_u}{D\mathrm{d}\vartheta} = \frac{\mathrm{d}\chi}{D\mathrm{d}\vartheta}\cdot F_v - \frac{1}{R}\cos\chi\cdot F_w - f_u^{gy} - f_u^{n},\\
&\frac{\mathrm{d}F_v}{D\mathrm{d}\vartheta} = \frac{1}{R}\sin\chi\cdot F_w - \frac{\mathrm{d}\chi}{D\mathrm{d}\vartheta}\cdot F_u - f_v^{gy} - f_v^{n},\\
&\frac{\mathrm{d}F_w}{D\mathrm{d}\vartheta} = \frac{1}{R}\cos\chi\cdot F_u - \frac{1}{R}\sin\chi\cdot F_v - f_w^{fr} - f_w^{gy},\\
&\frac{1}{R}\cos\chi\frac{\mathrm{d}\chi}{D\mathrm{d}\vartheta} = \frac{(A-C)}{A}\cdot\frac{1}{R}\cos\chi\frac{\mathrm{d}\chi}{D\mathrm{d}\vartheta} + \frac{F_v}{A},\\
&-\frac{1}{R}\sin\chi\frac{\mathrm{d}\chi}{D\mathrm{d}\vartheta} = \frac{(C-A)}{A}\cdot\frac{1}{R}\sin\chi\frac{\mathrm{d}\chi}{D\mathrm{d}\vartheta} - \frac{F_u}{A},\\
&\frac{\mathrm{d}}{D\mathrm{d}\vartheta}\left(\frac{\mathrm{d}\chi}{D\mathrm{d}\vartheta}\right) = -\frac{m_w^{fr}}{C}.
\end{aligned} \tag{3}$$

where f_u, f_v, f_w the external distributed forces (in *x*, *y*, and *z* directions respectively) should be determined; f_u^{gr}, f_v^{gr}, f_w^{gr} the known gravity forces; f_w^{fr} the friction force, and F_u the component of internal force vector.

In deriving Equations (3) it was taken into account that for the tubular drill string $A = B$, and the distributed points m_u and m_v equal to zero. It also was considered that $\vec{f}$ vector of external distributed forces acting on each element of the column consists of the gravitation force vector $\boldsymbol{f}^{gy}$, vector of the normal force or contact interaction of the column wells with well surface $\boldsymbol{f}^{n}$ and friction force vector $\boldsymbol{f}^{fr}$, *i.e.*

$$\boldsymbol{f} = \boldsymbol{f}^{gy} + \boldsymbol{f}^{n} + \boldsymbol{f}^{fr}. \tag{4}$$

The components of these forces are obtained by projecting the left and right sides of Equation (4) on axes u, v, w in the form of ratios

$$f_u = f_u^{gy} + f_u^{n},\quad f_v = f_v^{gy} + f_v^{n},\quad f_w = f_w^{gy} + f_w^{fr}, \tag{5}$$

where the gravitational forces are calculated as follows

$$f_u^{gy} = \gamma g n_z,\quad f_v^{gy} = \gamma g b_z,\quad f_w^{gy} = \gamma g \tau_z, \tag{6}$$

Here γ is the string mass per unit length; $g = 9.82\ \text{m/s}^2$, while f_u^{n}, f_v^{n}, f_w^{fr} become the new searched variables. That's why the system of six Equations (3) contains eight unknowns F_u, F_v, F_w, χ, f_u^{n}, f_v^{n}, f_w^{fr}, m_w^{fr} and becomes uncertain. *n*: is the unit vector in *z*-axis, and *b*: is the binomial vector.

After introduction of designations $\chi = h_1$, $\frac{\mathrm{d}\chi}{\mathrm{d}\vartheta} = \frac{\mathrm{d}h_1}{\mathrm{d}\vartheta} = h_2$ and performance of relevant transformations we bring system (3) to the following form

$$\frac{\mathrm{d}h_1}{\mathrm{d}\vartheta} = h_2,\quad \frac{\mathrm{d}h_2}{\mathrm{d}\vartheta} = \frac{1}{D}\cdot\frac{\mathrm{d}D}{\mathrm{d}\vartheta}\cdot h_2 - \frac{D^2 m_w^{fr}}{C}, \tag{7}$$

$$F_u = \frac{C}{R}\sin h_1\cdot\frac{h_2}{D},\quad F_v = \frac{C}{R}\cos h_1\cdot\frac{h_2}{D}, \tag{8}$$

$$\frac{\mathrm{d}F_w}{D\mathrm{d}\vartheta} = \frac{1}{R}\cos h_1\cdot F_u - \frac{1}{R}\sin h_1\cdot F_v - f_w^{fr} - f_w^{gy}, \tag{9}$$

$$f_u^n = \frac{\sin h_1 \cdot m_w^{fr}}{R} + \frac{C}{R^2} \cdot \frac{\mathrm{d}R}{\mathrm{d}\vartheta} \cdot \sin h_1 \cdot \frac{h_2}{D^2} - \frac{1}{R} \cos h_1 \cdot F_w - f_u^{gy}, \tag{10}$$

$$f_v^n = \frac{\cos h_1 \cdot m_w^{fr}}{R} + \frac{C}{R^2} \cdot \frac{\mathrm{d}R}{\mathrm{d}\vartheta} \cdot \cos h_1 \cdot \frac{h_2}{D^2} + \frac{1}{R} \sin h_1 \cdot F_w - f_v^{gy}, \tag{11}$$

$$f_w^{fr} = \pm\mu \left[\sqrt{\left(f_u^n\right)^2 + \left(f_v^n\right)^2} \cdot \frac{\dot{w}}{\sqrt{\dot{w}^2 + \left(\omega \mathrm{d}/2\right)^2}} \right], \tag{12}$$

$$m_w^{fr} = \pm\mu \left[\sqrt{\left(f_u^n\right)^2 + \left(f_v^n\right)^2} \cdot \frac{\mathrm{d}}{2} \cdot \frac{\omega \mathrm{d}/2}{\sqrt{\dot{w}^2 + \left(\omega \mathrm{d}/2\right)^2}} \right]. \tag{13}$$

where $\dot{w}$ is the speed of the DS axial movement; ($\omega d/2$) is the speed of rotational movement of DS element. The peculiarity of the system of Equations (7)-(13) is that it is due to the formulation of inverse problems for some variables along with the differential Equations (7) and (9) there are Equations (8), (10), (11), (12), (13), acting as first integrals. Therefore, the numerical solution of this system uses a special approach, which consists in the fact that the solution is not to build from the beginning of $\vartheta = 0$ of the integration area, where functions h_1, h_2, F_w are unknown, but from the end of ϑ_s, at which during the descent-ascent operations $h_1 = 0$, $h_2 = 0$, $F_w = 0$. The integration of differential equations is carried out step-by-step according to Runge-Kutt's method.

3. Results and Discussion

Using the developed approach the influence of the type of well axial line outlines on the value of static parameters of the deformed DS status upon descent-ascent operations. The case when the coordinates of the start and end points of the well coincide, and its axial line is located in the vertical plane passing through these points was considered. The simplest laws of their geometry (**Figure 1**) in the form of ellipse arches (1), parabola (2) and hyperbolas (3, 4) were selected.

The start and end points of all these curves coincide, however, between these points the mentioned curves have a shape with different curvature. A characteristic feature of the ellipse arc (curve 1) is that the tangents to it at points $x = 0$

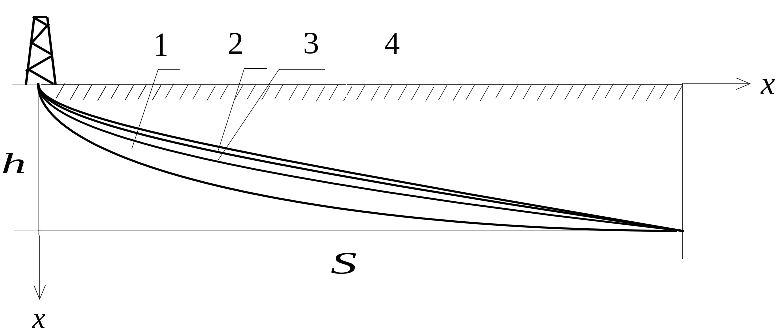

Figure 1. Geometrical layout of wells of the elliptical (1), parabolic (2) and hyperbolic (3, e = 1.01; 4, e = 1.015) shapes ($h = 1000м$, $S = 4000м$) {where the ellipse equation is $S^2/a^2 + h^2/b^2 = 1$, parabola: $4pS = h^2$, hyperbola: $S^2/a^2 - h^2/b^2 = 1$, a, b, and p are constants}.

and $x = S$ are vertical and horizontal respectively, and its curvature, though being maximal in point $x = 0$, has comparatively small values all along the interval $0 \le x \le S$. In this regard, we can assume that all the functions of the external and internal forces are distributed in such a drill string more evenly. The parabolic trajectory (curve 2) placed above it has smaller curvature over most of the range $0 \le x \le S$, but upon approximation to point $x = 0$ its curvature is increased and starts exceeding the curvature of the elliptical curve significantly. Therefore, we can assume that in the side area of the well the DS is affected by frictional and contact interaction forces with greater values. The peculiarities marked for parabolic hole in an even greater extent are shown for hyperbolic trajectories (curves 3, 4). With the selected values of geometric parameters with increasing of x these curves asymptotically approach the straight lines, but in the vicinity of the edge $x = 0$ the hyperbolic curves acquire material deviation.

Above it was assumed that the parameter affecting the critical state of DS in the performance of descent-ascent operations is a function of the curvature of the axial line of the channel well within small values ϑ. To confirm this assumption, let's present a table of values of the radii of R curvatures in the start and end points of the axial line of the well (see Table 1 for the case of $h = 2000м$, $S = 8000м$).

R is calculated by using the equation:

$$\frac{1}{R} = \sqrt{\left(x''\right)^2 + \left(y''\right)^2 + \left(z''\right)^2}, \tag{14}$$

It shows that upon transfer from an elliptic curve to a hyperbolic one the R value in point $\vartheta = 0$ decrease rapidly, which can be considered as the course for increase of internal and external forces acting on the string, and the reason for deterioration of the descent-ascent operations deterioration. Nonetheless, upon approximation of ϑ to $\vartheta = \vartheta_S$ the noted pattern becomes inverse; however, it has practically no impact on the conditions of axial movement of the drill string inside the well channel.

4. Conclusion

Based on the theory of flexible curved rods, the direct and inverse problems of bending deformation of the drill strings in the channels of curve wells are formulated. The problems of determination of the resistance forces and their momentums during the descent-ascent operations in the curved wells with trajectories

Table 1. Valued of R curvatures radii in the start and end points of the well axial line.

	Curve type	$R(м)$	
		$\vartheta = 0$	$\vartheta = \vartheta_S$
1	Ellipse	500	32,000
2	Parabola	250	131,012
3	Hyperbola (e = 0.01)	170	289,586
4	Hyperbola (e = 0.015)	129	504,969

in the form of curves of the second order are solved. The investigation of sensitivity of the resistance forces acting on the drill string during the descent-ascent operations with respect to geometric parameters of the trajectory of the well axial line was carried out.

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